Integer Linear Programming Model with Discretized Time Horizon for Solving Alternative Machine Scheduling Problems on Single Operation

Irwan Sukendar
Industrial Engineering Department
Universitas Islam Sultan Agung, Semarang 50112, INDONESIA
Phone.+62-24-658 3584, Fax.+62-24-658 2455, Email : irwan0203@yahoo.com

***Abstract*:** Some researchers discuss alternative machine scheduling problems. To solve the problems they develop integer linear programming (ILP) model base on disjunctive constraint formulation. In this paper, I am interesting to solve the problems with developing integer linear programming (ILP) base on discret time horizon formulation. Method of the research is : studying literature, developing model, and applicating into numerical example. The result is ILP model with dicretized time horizon. The model is usefull for solving alternative machine scheduling problems on single operation.

***Keywords** : scheduling, alternative machine, ILP, discrete time horizon*

1. INTRODUCTION

Scheduling is allocation resources to do some jobs on defined time [Baker, 1997]. We can solve scheduling problems with formulating problems into integer linear programming (ILP) formulation. There are two versions of ILP formulations : disjunctive constraint formulation and discretized time horizon formulation. [Morton and Pentico, 1993]

One of flexibility in manufacturing system is process flexibility. It is ability of manufacturing system to produce same product type in different way. There are some alternative routings or machines to do it. [Halim and Chandrawijaya, 1996].

In this paper, I am interesting to discuss ILP model with discretized time horizon for solving alternative machine scheduling problem on single operation.

Alternative machine scheduling problems are different from classic job shop problems. Each operation can be processed on association of alternative machines.

It can be defined as association of H machines and association of I jobs. Some machines of H machines can process each job. But only one of all of the association of H machine process the job. [Scrich, 2004]

The aim of this paper are : Developing Integer linear programming model with discretized time horizon for solving alternative machine scheduling problem on single operation, then applicating the model into numerical example.

2. THEORITICAL BASE

2.1. Weighted Tardiness Function

In scheduling problem with weighted tardiness function, each job has due date and weight. If completion time is older then due date, the job is got pinalti that is represented by the weight.

Tardiness is defined as difference of completion time from due date if completion time is older than due date.

Notation :

T_i = tardiness job i,
D_i = due date job i,
C_i = Completion time job i.

$$T_i = \begin{cases} C_i - D_i, C_i > D_i \\ 0, \quad C_i \le D_i \end{cases} \quad (1)$$

$$T_i = \max\{0, C_i - D_i\} \quad (2)$$

So that, the function is not linear.

Morton dan Pentico (1993) combine *Earliness* to *tardiness* function :

$$T_i - E_i = C_i - D_i \quad (3)$$

If $C_i > D_i$, so that $T_i = C_i - D_i$ (4)

If $C_i \le D_i$, so that $E_i = D_i - C_i$ (5)

Adding earliness variabel changes the function so that the function becames linear.

2.2. Scheduling Model Based on Descretized Time Horizon

2.2.1. Scheduling model from Morton and Pentico

Scheduling model from Morton and Pentico [1993], is a descretized time horizon based scheduling model. Some assumption of the model are : dynamic machine availability, dynamic job arrivals, and weighted flowtime objective function.

Notation :

C_{ih} = completion time job i on machine h
C_{ihk} = 1 if $C_{ihk} = k$; 0 if not.
x_{ihk} = 1 if job i scheduled on machine h slot k; o if not.
P_{ih} = processing time job i on machine h
T = total number of time slot
a_h = availability time on machine h

Model Formulation

Formulation of objective function :

$$Minimize \sum_{i=1,N} w_i C_{ih(i)} \quad (6)$$

Formulation of constraint function :

a. $C_{ih} = \sum_{k=1,T} kC_{ihk}$ for all (i,h) (7)

b. $x_{ihk} = \sum_{l=k,k+t_{ih}-1} C_{ihl}$ for all $(i,h),k$ (8)

c. $C_{ih} \ge C_{ig} + P_{ih}$ for all (i,h) (9)

d. $\sum_i x_{ihk} \le 1 \quad (i \in \{(i,h)\})$ for all (h,k) (10)

e. $x_{ihk} = 0, \quad k < a_h$ for all (i,h) (11)

f. $C_{ihk} \in \{0,1\}$ for all $(i,h),k$ (12)

2.2.2. Scheduling Model from Suprayogi and Toha

Scheduling model from Suprayogi and Toha [2002] is scheduling model for simultaneously resources. Suprayogi and Toha built the model based on discretized time horizon.

Model Definition and Assumptions :

There are N jobs and H kind of resources. There are only one operation in each job. D_i explains due date for each job. Each operation j for job i needs processing time t_i. H_{ij} explains association of resources process operation j job i. Time horizon is discretized on K time slots. On slot k, each kind of resource h $(h \in H)$ has availability M_{hk}.. w_i explains weight of each job. Operation process is nonpreemptive. Processing time for each operation is same for each resource. Each unit for each job is identic. All jobs are ready in $k = 1$. Time horizon k is enough to complete all jobs.

Notation :

N = association of jobs
H = association of resource
H_i = association of resource used by job i.($H_i \subseteq H$, $H_i \neq \emptyset$)
K = long of time horizon
M_{hk} = availability constanta to resource h on time slot k
t_i = processing time job i ($t_i > 0$)
w_i = weight of job i
D_i = due date job i
B_i = starting time to process job i
C_i = completion time for job i
b_{ih} = starting time to process job i on resource h
c_{ih} = completion time for job i on resource h
c_{ihk} = binary variable; $c_{ihk} = 1$ if job i that processed on resource h is completed on slot k, $c_{ihk} = 0$ if other
x_{ihk} = binary variable; $x_{ihk} = 1$ if job i is processed on resource h on slot k, $x_{ihk} = 0$ if other

Model Formulation

Formulation of objective function

The objective function if minimization of *total weighted tardiness*

$$Minimization \quad Z = \sum_i w_i T_i \quad (13)$$

Formulation of constraint resource :

a. $C_i - D_i - L_i + E_i = 0, \ \forall i$ (14)

b. $C_i \ge t_i, \ \forall i$ (15)

c. $c_{ih} = C_i, \ \forall i, h; h \in H_i$ (16)

d. $c_{ih} = \sum_k kc_{ihk}\ ,\ \forall i,h;h \in H_i$ (17)

e. $x_{ihk} = \sum_{k \le l \le k+t_{ij}-1} c_{ihl}\ ,\ \forall i,h,k;h \in H_i$ (18)

f. $\sum_k c_{ihk} = 1,\ \forall i,h;h \in H_i$ (19)

g. $\sum_i X_{ihk} \le M_{hk}, \forall h,k;h \in H_i$ (20)

h. $c_{ihk} \in \{0,1\},\ \forall h,k;h \in H_i$ (21)

i. $B_i = C_i - t_i + 1, \forall i$ (22)

j. $b_{ih} = b_i, \forall i, h \in H_i$ (23)

3. MODEL DEVELOPING

3.1. Description

In this paper, we duscuss alternative machine scheduling. It can be explained : there is association of job I. Every job $i \in I$ have due date D_i and weight w_i. H explains association of alternative machines. $H_{ij} \subseteq H$, $H_{ij} \neq \emptyset$ explains association of alternative machines that can process operation j on job i. Each job i can be processed on one of machine $h \in H_{ij}$. Operation time job i on machine h is explained by matrix t_{ijh}. Time horizon is descretized in K time units.

If there are 6 jobs 4 machines. Due date and weight are defined for each job. Job 1 and job 4 consist 2 operations, while the other jobs consist 1 operation. Time horizon is descretized in 8 time units. The problem can be saw in table 1.

Table 1. Problem description

Job	Operation time for each machine (days)				Due date (days)	Weight	Time horizon (days)
	1	2	3	4			
1	t_{111}	t_{112}	-	-	D_1	W_1	K
2	t_{211}	t_{212}	-	-	D_2	w_2	
3	-	-	t_{313}	t_{314}	D_3	w_3	
4	t_{411}	t_{412}	-	-	D_4	W_4	
5	t_{511}	t_{512}	-	-	D_5	w_5	
6	-	-	t_{613}	t_{614}	D_6	w_6	

3.2. Notation

1. Associaton
   H_i= association of alternative machine to process job i.
2. Index
   i = job
   h = machine
   k = time slot
3. Parameter
   N = number of job
   W_i = weight of job i
   D_i = *Duedate* job i
   K = time horizon
   t_{ih} = time to process job i on machine h, $h \in H_i$
4. Decision variable
   C_i = completion time job i.
   C_{ih} = completion time job i on machine h
   X_{ih} = biner variable 0-1 that explain job i is processed by machine h
   C_{ihk} = biner variable 0-1, 1 if job i that is processed by machine h ended on time slot k, 0 if not.
   T_i = *Tardiness* on job i
   E_i = *Earliness* on job i
   t_i = operation time on job i
   X_{ihk} = biner variable 0-1, 1 if job i on time slot k use machine h, 0 if other
   B_i = starting time on job i.
5. Performance measure
   Z = total weighted tardiness

3.3. Asumption

Asumption of the model are :

1. All jobs are ready in time slot k=1
2. Operation time includes set up time.
3. Processing of job is nonpreemptive.
4. Time horizon K is enough to process all jobs, $C_i \le K$ for all of i.

3.4. Model Formulation

Formulation of objective function :
Objective function is minimize total weighted tardiness

$$\text{Minimize } Z = \sum_i w_i T_i \quad (13)$$

Formulation of constraint function:

a. Constraint of tardiness function.

$$T_i - E_i - C_i + D_i = 0, \forall i \quad (14)$$

b. This constraint explains that job completion time must be same or older than the job operation time.

$$C_i \ge t_i, \forall i \quad (15)$$

c. Constraint of job operation time

$$t_i = \sum_h t_{ih} X_{ih}, \forall i; \forall h; h \in H_i; \quad (16)$$
$$H_i \subseteq H; H_i \neq \phi$$

d. This constraint guarantees that each job is processed by only one machine

$$\sum_h X_{ih} = 1, \forall i, \forall h, h \in H_i; \quad H_i \subseteq H, H_i \neq \phi \tag{17}$$

e. Constraint of job completion time on machine h. This constraint explains that completion time of job *i* on machine *h* must be same or older than operation time of the job on machine *h*.

$$C_{ih} \geq t_{ih}, \forall i, \forall h; h \in H_i; \quad H_i \subseteq H; H_i \neq \phi \tag{18}$$

$$\sum_h C_{ih} = C_i, \forall i, \forall h; h \in H_i; \quad H_i \subseteq H; H_i \neq \phi \tag{19}$$

f. This constraint defines job completion time on each machine. Because c_{ihk} = 0, except in completion time, so that adding on right side = *k* x 1, where is *k* is completion time

$$\sum_k C_{ihk} k = C_{ih}, \forall i, \forall h; \quad h \in H_i; H_i \subseteq H; H_i \neq \phi \tag{20}$$

g. This constraint guarantees that job *i* on machine *h* is completed once for time horizon.

$$\sum_k C_{ihk} = 1, \forall i, \forall h; h \in H_i; \quad H_i \subseteq H; H_i \neq \phi \tag{21}$$

h. This constraint guarantees only one machine processes job *i*

$$X_{ih} \in \{0,1\}, \forall i, \forall h; h \in H_i; \quad H_i \subseteq H; H_i \neq \phi \tag{22}$$

i. Binary constraint for decision variable C_{ihk}

$$C_{ihk} \in \{0,1\}, \forall i, \forall h, \forall k; \quad h \in H_i; H_i \subseteq H; H_i \neq \phi \tag{23}$$

j. Machine constraint. This constraint guarantees if job is completed in time slot *k*, so that between time slot (k-t_i+1) until *k* , the job is processed by machine *h*.

$$\sum_{k \leq l \leq k+t_{im}-1} C_{ihl} = X_{ihk}, \forall i, \forall h, \forall k; \quad h \in H_i; H_i \subseteq H; H_i \neq \phi \tag{24}$$

k. Machine availability constraint in each time slot. This constraint guarantees that machine using may not exceed machine availability..

$$\sum_i X_{ihk} \leq H_{hk}, \forall h, \forall k; \quad h \in H_i; H_i \subseteq H; H_i \neq \phi \tag{25}$$

l. Binary constraint for decision variable X_{ihk}

$$X_{ihk} \in \{0,1\}, \forall i, \forall h, \forall k; \quad h \in H_i; H_i \subseteq H; H_i \neq \phi \tag{26}$$

m. Constraint for processing starting time.

$$B_i = C_i - t_i + 1, \quad \forall i \tag{27}$$

4. NUMERICAL EXAMPLE AND RESULT

The model is applicated into this numerical example.

Table 2. Numerical example

Job	Operation time for each resource (days)						*Due date* (days)	weight	Time Horizon (days)
	1	2	3	4	5	6			
1	3	4	3	7	5	6	3	1	12
2	5	6	4	4	3	2	3	5	
3	5	3	4	6	7	5	3	2	
4	6	7	3	4	6	5	3	3	
5	7	7	6	5	4	3	3	3	
6	3	7	5	6	3	4	3	4	
7	7	4	4	5	6	3	6	4	
8	5	7	3	4	5	6	6	2	
9	7	6	5	3	4	6	6	5	
10	7	7	6	3	5	4	6	1	

Table 3. Scheduling solution

Re-source	Time horizon											
	1	2	3	4	5	6	7	8	9	10	11	12
1		6			1							
2		3										
3		4			8							
4		9			10							
5		2										
6		5			7							

The solution of minimization total weighted tardiness is 3 days

5. CONCLUSIONS

1. This paper produces result integer linear programming model with descretized time horizon for solving alternative machine scheduling on single operation.
2. The model was aplicated into numerical example 10 jobs 6 resources. The solution of minimization total weighted tardiness is 3 days.

REFERENCES

Baker,K.R, 1997, *Introduction to sequencing and scheduling, Dartmount College.*

Bedworth,D, Bailey,J, 1987, *Integrated Production Control Systems, Jon Wiley & Sons.*

Morton,T.E, Pentico,D.W, 1993, *Heuristic Scheduling Systems*, John Wiley & Sons.

Kusiak,A, 1990, *Intelligent Manufacturing Systems*, Prentice Hall, New Jersey.

Ladsaria,L , *Local and Global Scheduling*, http://ladsaria@uiuc.edu

Sipper,D, Bulfin,R, 1997, Production Planning, Control, and Integration, Mc-Graw-Hill.

Suprayogi, Toha,I.S, 2002, Model Pemrograman Linier Bilangan Bulat untuk Masalah Penjadwalan Sumber Majemuk Paralel Simultan, Jurnal Teknik dan Manajemen Industri ITB, volume 22 nomor 1, April.

Vollmann,T.E, Berry,WL ,Whybark,DC, 1988, *Manufacturing planning and control system*, Dow jones-irwin, Homewood, Illinois.

AUTHOR BIOGRAPHIES

Personality

Name	: Irwan Sukendar
Date of Birth	: Semarang, 10 Januari 1976
Sex	: Male
Religion	: Islam
Nationality	: Indonesia
Address	: P4A Blok F/14 RT.7 RW.IX Pudakpayung Semarang
Phone	: 024-70426809
Email	: irwan0203@yahoo.com

Academics

1. Industrial Engineering Department, Indonesian Islamic University (UII) Yogyakarta, Undergraduate (S1) in 1998
2. Industrial Engineering Department, Bandung Institute of Technologi (ITB), Graduate (S2) in 2006

Researches

1. Analisis Peningkatan Mutu Produk Susu Full Cream Milk Powder dengan menggunakan Metoda Quality Function Deployment - Studi Kasus pada PT. Sari Husada Yogyakarta (Diterbitkan pada Jurnal ilmiah ISSN 'TRANSISTOR' – Fakultas Teknol;ogi Industri Universitas Islam Sultan Agung Semarang, September 2001.
2. Perencanaan Kebutuhan Material pada Konstruksi Bangunan dengan Menggunakan Metode Integer Linear Programming (Diterbitkan pada 'PROCEEDING' ISBN Seminar Nasional Manufacturing System, Surabaya, 1Oktober 2001)
3. Membangun Perekonomian Nasional Melalui Sistem Agroindustri Menyeluruh (Diterbitkan pada 'PROSIDING' ISBN Seminar Nasional Teknik Industri III, Surakarta, 30-31 Juli 2002)
4. Analisis dan Perbaikan Sistem Antrian untuk Mengurangi Waktu Menunggu (diterbitkan pada Jurnal ilmiah ISSN TRANSISTOR – Fakultas Teknologi Industri UNISSULA Semarang, Maret 2003)
5. Analisis Penjadwalan Produksi pada Mesin Produksi Kertas (diterbitkan pada PROCEEDING ISBN Seminar Nasional "Pengaruh Perubahan Paradigma Bisnis dan Industri terhadap Kompetensi Teknik Industri", Semarang, 31 Mei 2003)
6. Analasis Sistem Perawatan Peralatan Produksi PT. Coca Cola Pan Java (Jurnal ilmiah terakreditasi TRANSISTOR, Juli 2006)