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Construction of Students' Mathematical Knowledge in the Zone of Proximal Development and Zone of Potential Construction

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Abstract: This article highlights the main ideas that underlie the differences in potential pragmatic knowledge constructs students experience when solving problems, between the zone of proximal development (ZPD) and the zone of potential construction (ZPC). This qualitative research is based on a phenomenological approach to finding the meaning of things that are fundamental and essential from the ZPD and ZPC phenomena. Researchers observed mathematics learning by a teacher on 24 fourth-grade students who were divided into groups A (high IQ) and B (low IQ). Data collection through tests, observation, and interviews. While the validity of the data is done through triangulation of methods and triangulation of sources. The results showed that students of the Upper (A) group had high IQ but small ZPD and ZPC. In contrast, students in the Lower (B) group have low IQ but large ZPD and ZPC. This result means that intelligence (IQ) is measured not only logically-mathematically but also in the verbal-linguistic and spatial-visual fields. The conclusion is that there are differences in the construction of students' knowledge in the learning zone. This difference occurs because the knowledge constructs that the students have previously had an effect on the accommodation process of the schemes that students have built while in the proximal development zone (ZPD) where scaffolding works. Meanwhile, the potential construction zone (ZPC) is not sufficient to describe the real development of students. However, it only reflects what students have accomplished.

Keywords: *Construction of knowledge, zone of proximal development, zone of potential construction, scaffolding.*

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Introduction

Mathematics as a human activity is a theory put forward by Hans Freudenthal (1905-1990). The meaning is that mathematics is a human activity where students are given an opportunity to learn in mathematical activities and can find mathematical ideas or create models of students' thinking (Gravemeijer & Terwel, 2000). In this process, in order for thoughts to lead to teaching mathematics, it is necessary to provide assistance to students. Thus, many opportunities are given by teachers to students to build their own understanding, starting from activities that are known and understood by students.

Constructivist ideas have been widely applied by researchers as the rationale for various cooperative learning techniques to facilitate students' knowledge construction. There are two different views about the knowledge construction of learners in learning. First, based on Rieber & Carton's (1987) theoretical framework which describes the social construction of knowledge. Second, the radical constructivist theory of von Glasersfeld (1995) which explains schema construction and cognitive operations (von Glasersfeld, 1995; Moll, 1990; Norton & D'Ambrosio, 2008; Shabani, 2016). However, both perspectives agree that the process of knowledge construction and its development is the result of the interaction of students with the social environment or other people (Jalilifar et al., 2017; Jamalinesari & Rahimi, 2015; Tinungki, 2019).

Educators assume that teaching becomes effective is when students can reach a set development threshold level, that is, when students can perform certain tasks independently (Darling-Hammond et al., 2020). Of course this assumption contradicts the opinion of Rieber and Carton (1987), that an educator cannot fully understand the level of development of children without determining the upper limit of this development through the types of tasks that children can do

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with the help of others (Walshaw, 2017). Two children may show the same grade on a task at which they have reached the same level of development and are in the same learning readiness. However, one of them may potentially be able to perform other, more complex tasks with the same assistance under the guidance of an adult (teacher).

On the other hand, the application of learning in mathematics classes where too often teachers make connections and show patterns for students unwittingly has taught them to imitate what teachers do rather than construct meaning for students themselves. In fact, time will be more effective if the teacher spends designing and implementing activities to help students construct their own knowledge, which in turn will equip them for independent study.

This article highlights the main ideas that underlie the potential differences in pragmatic knowledge construction between the two learning zones that students experience when solving problems, namely the zone of proximal development and the zone of potential construction.

In epistemological theory, differences in the construction of knowledge constructed by students have triggered arguments whether they are in the zone of proximal development or the zone of potential construction (Steffe & D'Ambrosio, 1995). Therefore, it is very important to carry out experiments and in-depth analysis to investigate the construction of students' knowledge when solving mathematical problems. Teachers must have the ability to investigate and understand mathematical ideas developed by students (Esendemir & Bindak, 2019). The results of this investigation will be useful for teachers to design the provision of learning assistance to students so that students can achieve optimal cognitive development.

Theoretical Framework

Zone of Proximal Development

The Zone of Proximal Development (ZPD) is the distance between the child's current level of actual development in independent problem-solving activities and the potential level of development that the child can achieve in solving problems under the guidance of a teacher or adult, or a more competent peer (Abtahi et al., 2017; Eun, 2019; Moll, 1990; Rieber & Carton, 1987; Silalahi, 2019). ZPD consists of problems that a student might solve when "given a little help, solution step instructions, keywords, or guiding questions" (Rieber & Carton, 1987). This definition considers the development of the child, including the guidance of adults or friends who are more able to provide assistance in ZPD who demonstrate children's potential (Rieber & Carton, 1987; Walshaw, 2017).

ZPD defines functions that are immature or in the process of ripening. These functions will mature tomorrow, but are currently in an embryonic state (Petakos, 2018). The actual level of development characterizes mental development retrospectively, whereas the zone of proximal development characterizes mental development prospectively (Rieber & Carton, 1987). Rieber and Carton felt that action was needed to better measure the potential development of students. The ZPD conceptually provides an overview of how development potential can be implemented without measuring the IQ score (Fani & Ghaemi, 2011).

There are six elements that are important in ZPD, namely assistance, mediation, cooperation, ability to imitate, targets and difficult times or getting out of the comfort zone (Silalahi, 2019). The teacher's job is to facilitate learning, raise problems, and provide assistance when needed by students in ZPD. According to Rieber and Carton (1987, p.86) ".... the only kind of teaching that is good is that which is ahead of its development". So, to make progress to a higher level of development, students in learning need to be assisted, guided, and directed so that they can solve problems independently (Darling-Hammond et al., 2020). This raises the question: When can a teacher's help promote development? And what assistance should the teacher provide to ensure learning leads to math development? To answer these questions, the teacher must understand the cognitive development of students while studying in their ZPD area. If they point towards potential developments, then the teacher can provide scaffolding. Teachers must believe that every child has the potential to develop more advanced. Because basically there are no "stupid" children, only children who don't understand, so they need help.

Zone of Potential Construction

The zone of potential construction (ZPC) is referred to as the range determined by the modification of concepts that students may make (Steffe, 1991). Ranges describe the cognitive structures of students developing through the abstraction of actions and operations (Piaget, 1964). The formation of students' cognitive structures at ZPC, the construction can be provided by the teacher in the perspective of learning theory (Norton & D'Ambrosio, 2008; Steffe & D'Ambrosio, 1995; Steffe, 1991).

Epistemologically, there is a fundamental difference between social constructivists and radical constructivists (Norton & D'Ambrosio, 2008). Social constructivists view that the concept of problems and other forms of knowledge have existed previously in (the environment) society before being internalized by individuals (Rieber & Carton, 1987). Meanwhile, according to (von Glasersfeld, 1995) in the theoretical perspective of schemes, concepts, and knowledge, even society itself starts from experiences and unique individual constructs.

This perspective requires teachers to always pay attention to different conceptions or schemes that students have. Through a schema model built by students, teachers can find out about their progress. ZPC can be modeled by the teacher through the reorganization of existing schemes and operations in the student structure model, and assignment design must also depend on a specific student model (Steffe & D'Ambrosio, 1995).

Scaffolding

It is clear that help or guidance is needed in child development. The assistance in this ZPD is in the form of temporary scaffolding. In the ZPD function, scaffolding is most effective when it is tailored to the needs of students (Fani & Ghaemi, 2011; Moll, 1990).Within the ZPD function, assistance is most effective when it is tailored to the needs and development of students (Bikmaz et al., 2016; Brower et al., 2018; Kim & Belland, 2018; Silalahi, 2019). In the Vygotskian sense, the main feature of ZPD is its dialogical structure, where tutors and students engage in exchanges that aim to create consensus on the structure of the problem (task) at hand and the most appropriate action for the problem solution (Fani & Ghaemi, 2011; Mutekwe, 2018; Silalahi, 2019; Tinungki, 2019).

The scaffolding is a concept derived from cognitive psychology. That during social interaction students who are more capable through the use of language and other supporting conditions, can help children move forward to a higher level with their knowledge and skills (Darling-Hammond et al., 2020).The use of scaffolding that supports, facilitates, assists, and accelerates children's learning tasks is when teachers and peers use scaffolding in cooperative learning, and learning increases (Ghorbani, 2016).

ZPD and scaffolding are two concepts that can efficiently help a person learn a skill. Scaffolding involves experienced instructors who guide students through the tasks in their ZPD (Khaliliaqdam, 2014; van de Pol & Volman, 2019). The individual ZPD includes any task that can only be completed with assistance. When giving scaffolding, the goal is not to provide answers to students but to assist their learning with certain techniques such as encouraging, modeling, or giving directions (Pol & Beishuizen, 2015). When students begin to master a skill, the amount of support provided must be reduced (van de Pol & Volman, 2019).

Methodology

Research Method

This qualitative research is based on a phenomenological approach that will explore data to find the meaning of the facts and experiences experienced by the object of research (Khan, 2014). The focus of the phenomena to be studied is various subjective aspects of object (student) behavior when solving the problem of dividing fractions. Then the researchers extracted data in the form of how the object was interpreted in interpreting the related phenomenon.

Problem-based learning in this study, in its performance, applies the four stages of the zone of proximal development (Moll, 1990). Stage I: Demonstrating how students develop an understanding of the topic being studied by relying on the instructor to perform tasks. Stage II: Students use prior knowledge to complete the task without assistance. Stage III: Understanding is developed and knowledge is repaired automatically. At this stage students reach the independence stage. They do many exercises to strengthen existing knowledge. Stage IV: Students are in de-automation of performance which leads to a process of repeating the function, which is every time implementing the construction results from the previous stage through ZPD.

Participants

This study involved 24 participants in fourth-grade elementary school students, a class teacher, and two observers. The observer was a researcher and a final semester mathematics education student who was interested in this study. Its task is to observe learning interactions and provide feedback to teachers about the effectiveness of their learning. Based on the previously obtained IQ test, students were divided into two groups. Upper group (A) consisted of 12 students with high IQ scores, and the Lower group (B) consisted of 12 students with low IQ scores. The reason for grouping students based on this IQ level, as well as to find out whether students with low IQ will always be in the small ZPD and ZPC or vice versa? The performance of students in groups (A) and (B), each supervised by an observer. After the results of the students' work were analyzed, then the subjects (students) who were interviewed were selected through the purposive technique. Those who are nominated to be considered as sources of information are students who have communication skills and can give good answers. Finally, two students were chosen to represent the Upper group (A), namely the subject (S-A5; S-A7) and two students representing the Lower group (B), namely the subject (S-B14; S-B19).

Data collection and analysis

Data collection in this study was carried out by testing, conducting in-depth interviews with objects or informants in the study, and also the direct observation of how the research object behaves when completing the test and discussing interpreting its experiences to others (Daymon & Holloway, 2014). The results of the interview were analyzed as a basis for describing the knowledge construction of students, whether they were in ZPD or ZPC when solving the problem of dividing fractions. The research data obtained from observations and interviews in groups A and B were then tested for validity by triangulation. Method triangulation is done by comparing student work data and interview data. Source triangulation is done by checking the correctness of student work in groups A and B.

One of the problem items items that must be solved by students is as follows.

Problem	:	Mother has 2 banana cake pans and ½ banana cake pans. Mother will cut the whole cake into pieces.
Question	:	How many small pieces (¼ pieces) of cake do you get?

The problems in this test are related to problems in real-life every day that can be solved through logical reasoning or "intuitively" (Mula & Hodnik, 2020). The expected competence is that students can have mathematical reasoning skills in the material for dividing fractions. Mathematical reasoning is an important foundation in learning mathematics which must be started at the elementary school level (Damrongpanit, 2019).

Results

The implementation of mathematics learning in groups A and B were closely monitored. In order for learning activities to run well, the teacher gives each student the freedom to work on solving problems in their own way until they find the logical consequences of the problems formulated (Junarti et al., 2019). After the students did the test, it was obtained that 100% of the students in group A answered "correct". Meanwhile, 75% of students in group B answered "right", and 25% answered "wrong". During group discussions, student behavior was observed.

The results of the observation of group B students' learning activities were that they worked together, asked each other and discussed to complete the task. It seems that some students pay attention to the explanations of other students who are better able to complete the task. There are even students who have to be guided from the start by other students to complete their assignments.

Student activity in group B discussion shows the existence of peer-scaffolding, namely providing scaffolding support by utilizing the strengths of peers who are considered smarter or more capable in class. Empirical studies show that peer-scaffolding has a positive effect on cognitive outcomes and helps students successfully cope with problems (Kim & Belland, 2018). Peer-scaffolding that occurs in group B proves that group discussion is effective so that knowledge construction is in accordance with social construction (Steinbring, 2000). The following is the answer from the subject (S-B19) which is interesting to discuss as in Figure 1.

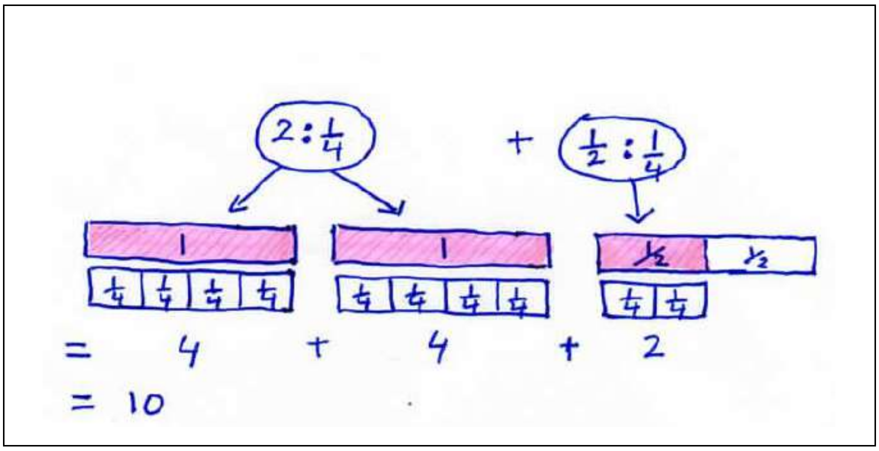

Figure 1. Subject Answers (S-B19)

Paying attention to the results of the subject's answers (S-B19) in Figure 1, the researcher (R) conducts a deeper search through interviews, so that researchers can describe the construction of subject knowledge into the appropriate learning zone.

1st Interview: ……………………………………………………………………

R	:	Why did you help and what did you do for your friend who was less able to complete the task?
(S-B19)	:	Because we're a team. I directed the subject (S.B14) to solve the problem in small parts.
R	:	Why do you illustrate the problem in picture form?
(S-B19)	:	This picture is to clarify the understanding of others.

R : Why did you break the problem into small pieces?

(S-B19) : I often do this activity at home when I share the cake. If there are a lot of cakes, we must cut them one by one.

(S-B14) : In my opinion, this method is easier for me to understand.

R : Give an explanation until you find the result.

(S-B19) : I'm ready to explain for the next steps.

The results of the observation of the learning activities of group A students were that they worked independently to complete the task. In general, they can quickly complete assignments given by the teacher. Learners practice on their own, which means that they are doing certain activities without assistance. However, they are not yet at the perfect proficiency stage and sometimes need help. Then after they have completed the assignment, they discuss together the results of their work. Here is one of the answers of the subjects (S-A7) which represents their group as shown in Figure 2.

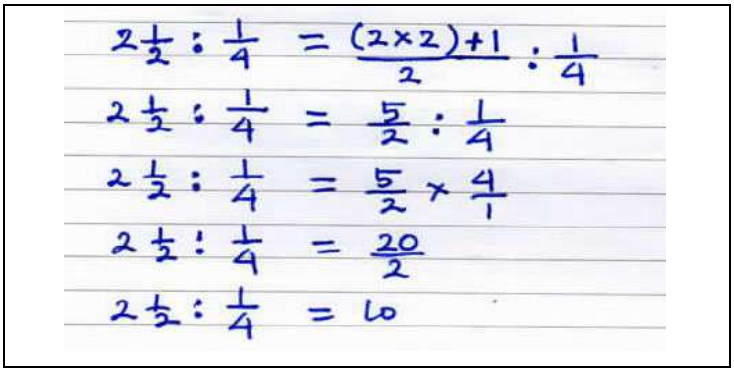

Figure 2. Subject Answers (S-A7)

The results of observing the students' answers, it turned out that all students in group A had a single solution (same answer) as in Figure 2. The researcher wants to describe the subject's knowledge construction into the appropriate learning zone. Next, the researcher (R) conducted a deeper search through interviews with one of the representatives, namely the subject (S-A7).

2nd Interview: ………………………………………………………………………

R : Are you sure about this completion step?

(S-A7) : Very confident. This fits with my planned completion strategy.

R : Why did you change the fraction 2 ½ to 5/2?

(S-A7) The idea is to divide by the fraction ¼

R : Can you explain why to solve the 5/2 problem: ¼ becomes 5/2 x 4/1?

(S-A7) : This is a quick way to solve the problem.

R : Do you know why the divisor must be reversed?

(S-A5) : I don't understand why it's like that.

(S-A7) : I just imitated what my previous teacher.

R : Is there any other way to solve this problem?

(S-A5) : I can't solve it any other way.

When the researcher asked for an explanation on the next step, the subject (S-B19) felt challenged by further questions from the researcher. Then, the subject (S-B19) provides a detailed explanation as shown in Figure 3 below.

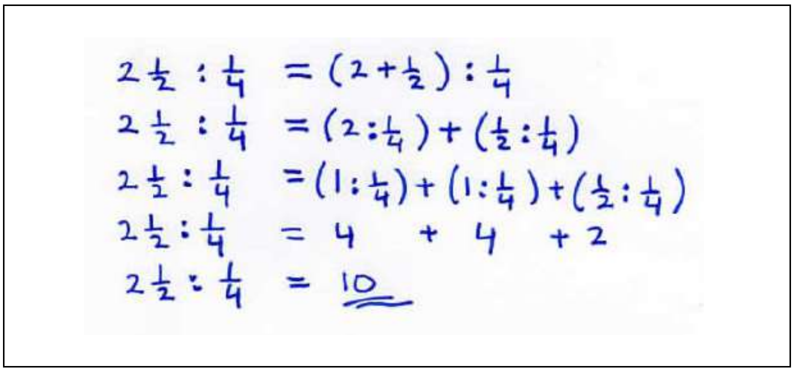

Figure 3. Subject Answers (S-B19)

Paying attention to the subject's explanation (S-B19) in Figure 3, the subject has been able to carry out an analytical thinking process, which is solving problems by breaking the problem into smaller parts (Wibawa et al., 2018). Researchers explore further to find out the knowledge built by the subject (S-B19).

3rd Interview: ……………………………………………………………………………

(S-B19) : This means 2 ½: ¼ = (2 + ½): ¼

R : Try to explain the results of your performance.

(S-B19) : This means 2 ½: ¼ = (2 + ½): ¼, then (2: ¼) + (½: ¼) and then broken down into smaller parts.

R : How many pieces of cake did you get?

(S-B19) : There are 10 pieces of cake.

R : In your opinion, the final calculation result is equal to 10, does it show an integer?

(S-B19) : Ten shows that there are 10 slices of cake, each of which is small, namely ¼ parts.

(S-B14) : I agree, ¼ looks smaller than before the cut cake.

R : What does the fraction ¼ mean?

(S-B19) : ¼ means one small part of four whole parts, or it can be written as 1: 4

Furthermore, to investigate the students' knowledge construction process (S.A5 and S.A7), the researcher conducted deeper interviews accompanied by a one-to-one scaffolding process.

4th Interview: ……………………………………………………………………

R : Would you like to find another, more logical way?

(S-A5) : Yes, for sure

(S-A7) : Yes, I am curious about that way.

R : Before you divide 5/2 by ¼, try multiplying 5/2 by the number 1 or another equivalent name, which is 2/2 or 3/3 or 4/4, etc.

(S-A7) : I chose (5/2 x 4/4)

(S-A5) : I chose (5/2 x 2/2)

R : Good choice.

(S-A7) : (5/2 x 4/4): ¼ = 20/8: 1/4

(S-A5) : (5/2 x 2/2): ¼ = 10/4: 1/4

R : Please calculate the result by dividing the numerator and the denominator divided by the denominator.

(S-A7)	:	Wow, ... the result is 20/2 = 10
(S-A5)	:	10/1 = 10
R	:	Why are you multiplying the number 5/2 by 2/2, or 4/4?
(S-A7)	:	Because, 2/2 or 4/4 is equal to 1. The nature of the multiplication operation by number 1 is that the result does not change the value of that number.

The results of the subject's performance in solving problems after receiving one-to-one scaffolding are presented in Figure 4 below.

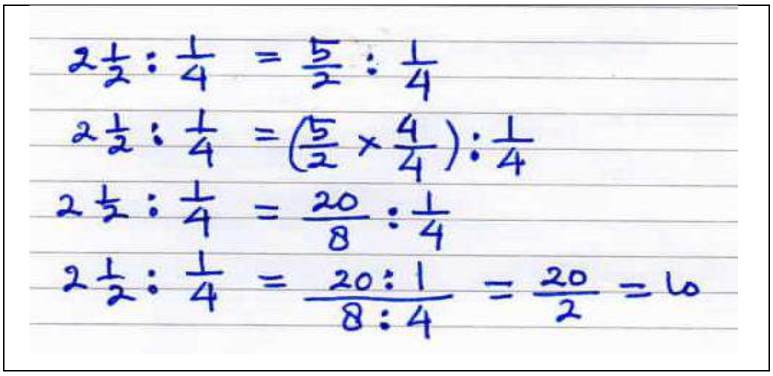

Figure 4. Subject Answers (S-A5)

Discussion

In the interview snippet above (1st Interview), it appears that the subject (S-B19) expresses himself by participating productively to advance group activities. Subjects (S-B14) (as participants who are considered less knowledgeable) also have an important role in generating ZPD (S-B19) in the mathematics learning environment. This means that in group learning there is a two-way interaction. Thus (ZPD) activities and knowledge construction depend together on the contribution and involvement of all participants (Abtahi et al., 2017; Breive, 2020). In addition, ZPD is not predetermined from activity, but appears nowadays as part of a shared object-oriented activity (Abtahi et al., 2017).

Subject (S-B19) has built a fraction division scheme on his ZPC. This step is demonstrated by a very clear illustrated image (see Figure 1). The mental operation that occurs on the subject's ZPC (S-B19) is related to determining a measure of equivalence by the repeated addition of ¼ parts (four times) to reproduce the whole that is 1 = ¼ + ¼ + ¼ + ¼. This shows that the ZPC subject (S-B19) is large, has also developed a fractional unit scheme that can be used to conceptualize fraction parts through addition relations (Norton & D'Ambrosio, 2008). The mathematical concept that was built by the subject (S-B19) in the division operation of the fraction turned out to already exist in its ZPC. Subject (S-B19) actualizes the new scheme through accommodation caused by the problem solving activity illustrated (Joubish & Khurram, 2011; Piaget, 1964).

During the interview (3rd Interview), the subject (S-B19) can show that the subject is working at the level of small units by breaking the problem into parts. Subject (S-B19) can also demonstrate solving the division of fractions and can interpret the fraction ¼ as a relation 1 part of 4 whole. Subject (S-B19) he has developed a scheme, which means that 10 pieces of cake are 10 pieces of cake, each of which is small, namely ¼ parts. Subject (S-B19) has developed a partial fraction scheme, and an equivalent fraction scheme (which can be used to generate and describe fractions of equal size) is already in his ZPC (Steffe & D'Ambrosio, 1995). The researcher makes this claim because we see the potential for the reorganization of knowledge through the accommodation process of his partitive fraction division scheme into smaller parts. Problem-solving activities carried out by students produce new construction models and how to operate them.Whereas in the process of solving the subject's problem (S-B19) using conceptual knowledge that are constructed by themselves (Österman & Bråting, 2019).

Paying attention to the interview excerpt (2nd Interview), the partial fraction scheme that can be reversed from the division operation to multiplication, and ¼ to 4/1 already exists in the subject's ZPD (S-A7), but is still outside of its ZPC. In fact, the subject actualizes the new scheme through the accommodation caused by the problem-solving activity created. However, when the researcher asked to explain, the subjects (S-A5; S-A7) experienced difficulties, and could not provide a reason why the divisor should be reversed. So that the researcher argues that the subject (S-A7) only uses procedural knowledge to solve problems (Österman & Bråting, 2019).

Based on snippets of interviews with the subject (S-A7), the researcher concluded that the subject's knowledge construction was obtained from the ability to imitate (Silalahi, 2019). The subject imitated the steps that the teacher had conveyed in the previous lesson. They cannot explain the reason, why the problem in the fraction division operation is solved by a multiplication operation where the divisor, that is reversed to be 4/1. The performance of students entering stage IV, which is in the de-automation of performance which leads to a repetition of the function, namely every time applying it to the results of the previous stage through ZPD. So it can be said that the subject (S-A7) has a small ZPC. However, because the final solution has the correct answer, the subject feels comfortable with this understanding, as a result, the subject is difficult to get out of the ZPD (Silalahi, 2019).

The researcher assumes (4th Interview and see Figure 4) that when the subject (S-A5; S-A7) is in one-to-one scaffolding (Kim & Belland, 2018), the knowledge construction lies in the ZPC, where its operation is challenged by the way the researcher raises the accommodation of the scheme and can be completed correctly. This accommodation refers to the mental process in which learners actively rearrange plans (conflicts) contained in their experiences (Liang, 2019; Piaget, 1964). If, the subject's performance is not challenged by giving rise to the accommodation of the scheme, then the subject (S-A7) does not work at his ZPC (Steffe & D'Ambrosio, 1995).

Researchers believe that the subject (S-A5; S-A7) is able to demonstrate an understanding of this fractional operation problem. We noticed that after the subjects (S-A5 and S-A7) followed the direction during the scaffolding, there appeared to be progress (see Figure 4). One-to-one scaffolding from teachers to students has succeeded in deconstructing students' thinking structures so that students can achieve ZPD (Wibawa et al., 2018). Subjects (S-A5; S-A7) have a large ZPD with more constructing new knowledge. As Rieber and Carton (1987, p.188) said, "Optimal learning can emerge when each student progresses in ZPD". All children (learners) have the possibility to get something different or build all the abilities that exist. In this zone, children (students) feel motivated, excited, and challenged to learn more deeply (Darling-Hammond et al., 2020). In fact they were able to achieve more advanced developments, but why didn't they continue into the zone? because in previous lessons they did not get effective information and social service support (Jalilifar et al., 2017).

If the information conveyed in learning is outside the student's ZPD, it will pose a risk. First, the material is very straightforward, simple and unsophisticated; thus causing students not to accept challenges, get bored quickly and passively learn. Second, the subject matter is very high so students find it difficult to understand it. Both of these conditions will hinder learning and have the potential to become a failure, especially for students at the elementary school level (Suranata et al., 2018).

The teacher's actions when providing scaffolding on the subject (S-A5 and S-A7) aim to guide students to think as they do. This is important because (1) by continuing to build meaning, teachers can develop new hypotheses about learners' cognition; (2) the position of the teacher in order to understand how students (think) operate and compare it in their own way to design tasks that trigger students' creative activities.

Researchers compared the performance results of students in the upper group (A) and the lower group (B) that the performance levels of students would be helped in ZPD which highlighted the potential for emerging behaviors and future development. Subjects (S-A5; S-A7) may have high IQ but small ZPD and ZPC, and subjects (S-B14; S-B17) may have low IQ but large ZPD and ZPC (Fani & Ghaemi, 2011).

It turns out that in this case, students (S-B19) with low IQ have good analytical and problem-solving skills. Figure 3 made by this student shows that students think through logical reasoning. Even analytically, the results of subject completion (S.B.19) in the form of images can be easily understood by other students.

This difference occurs because the construction of knowledge that students have previously had an influence on the accommodation process of the schemes that students built at what time in ZPD. This shows that intelligence is not only measured logically-mathematically, but also in the fields of verbal-linguistics, spatial-visual, musical, intrapersonal, naturalist, interpersonal, and existentialist (Kadwa & Alshenqeeti, 2020). Basically, all children have the opportunity to get something different or build all abilities in ZPD.

Thus, it can be said that the idea of ZPD is more meaningful than the learning situation presented to students, where teachers or peers who are more competent have a positive influence on other students (Mutekwe, 2018). Meanwhile, ZPC is not sufficient to describe the actual development of students. On the contrary, it only reflects what students have achieved or developed (Norton & D'Ambrosio, 2008).

Conclusion

The conclusion of this study shows that there are differences in ZPD and ZPC in student learning zones. ZPD of Upper group students (High IQ) is built on procedural knowledge obtained from previous learning outcomes, through a process of imitation, and is permanent. ZPD students in the Lower (Low IQ) group are built on conceptual knowledge, go through an analytical process, and are flexible. ZPC upper-class students (High IQ) are already in the concept of thinking but small or not strong enough. ZPC of students in the Lower (Low IQ) group as new knowledge, and big or strong. This difference occurs because the construction of knowledge that students have previously had an influence on the accommodation process of the schemes that students built while in ZPD. These results also show that intelligence is not only measured logically-mathematically but also in the verbal-linguistic and spatial-visual fields.

The idea of a zone of proximal development (ZPD) is more meaningful when there is scaffolding assistance (one-to-one scaffolding or peer-scaffolding) which has a positive effect on students' knowledge deconstruction. Meanwhile, the zone of potential construction (ZPC) is not sufficient to describe the real development of students. Rather, it only reflects what the student has achieved.

Suggestions

Mathematics is a subject that is quite difficult for students at the elementary school level. The gap between the abstract nature of mathematical objects and their level of cognitive development is still at a concrete stage, causing their learning difficulties.

On the other hand, educators (teachers) only look at the level of children's learning development on the surface. Teachers and parents often justify that children with test scores below the specified standards are classified as children with low understanding, considered unsuccessful (failing) the test, and not smart. This means that testing educators only looks at the level of achievement of student (child) learning development at this time, not the potential for developing student learning in the future.

In fact, every child has a zone of proximal development, children have the potential to develop towards optimal cognitive development, as long as they are directed and assisted by educators (teachers), adults (parents), or more competent peers. As a suggestion, teachers must optimize learning by monitoring student learning activities, assisting, and providing scaffolding to students who have difficulty solving problems, so that they can learn independently. The types of scaffolding that can be provided include one-to-one scaffolding, peer-scaffolding, or computer-based scaffolding.

Limitation

In this article the researcher makes the following limitations. (1) This research is a case study of students' knowledge construction in mathematics learning about the division of fractions in grade IV schools, so it is possible that in other cases learning mathematics with different grade levels will also be different construction of student knowledge; and (2) during the interview, students often could not provide answers (explanations) well, so the researcher had to repeat the questions and describe them according to a theoretical perspective, namely literary works.

Researchers who are interested in carrying out similar research can apply it to the object of research in upper class students (Junior High School, Senior High School, and Higher Education). Determination of the interviewed subjects can be selected through purposive and snowball methods, so as to provide a complete picture of the research results.

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