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ANALYSIS OF ABSTRACT REASONING FROM GRADE 8 STUDENTS IN MATHEMATICAL PROBLEM SOLVING WITH SOLO TAXONOMY GUIDE

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Abstract

This research was a descriptive research. Description of research result was presented quantitatively and qualitatively. Subjects of the research were 30 (thirty) 8th graders of SMPN 10 (State Junior High School) in Semarang, Indonesia. Data were collected through tests, documentation, observations, and interview. Student answers documents were observed and analyzed with SOLO Taxonomy guidance. The objective of the study was to analyze and provide an interpretation of students abstract reasoning level in cognitive development based on intended learning outcomes. The result of findings from students' answers basically showed that students' abstract reasoning on the lower, middle and upper level, was alike to stages of structure complexity improvement. There were two main changes from concrete thinking to abstract thinking: quantitative stage (*uni-structural* and *multi-structural*) occurred first, as the number of details in student responses increased and then changed qualitatively (*relational* and *extended abstract*) because the detail was integrated into a structural pattern.

Keywords: Abstract Thinking, Abstract Reasoning, Problem Solving, SOLO Taxonomy.

Abstrak

Penelitian ini adalah penelitian deskriptif. Deskripsi hasil penelitian disajikan secara kuantitatif dan kualitatif. Subjek penelitian sebanyak 30 siswa kelas 8 pada SMP Negeri 10 di kota Semarang, Indonesia. Pengambilan data melalui metode tes, dokumentasi, observasi dan wawancara. Respon jawaban siswa dalam tes penalaran abstrak diamati dan dianalisis dengan berpandu taskonomi SOLO. Tujuan penelitian untuk menganalisa dan memberi interpretasi tingkat penalaran abstrak siswa dalam perkembangan kognitif berdasar capaian pembelajaran yang diinginkan (*intended learning outcomes*). Hasil temuan dari tampilan respon jawaban siswa secara mendasar menunjukkan bahwa penalaran abstrak siswa pada kelompok bawah, tengah dan atas, serupa tahapan peningkatan kompleksitas struktur. Ada dua perubahan utama dari penalaran konkret menuju penalaran abstrak yaitu tahap kuantitatif (*uni-structural* dan *multi-structural*) terjadi pertama, seperti jumlah detail pada respon siswa meningkat dan kemudian mengalami perubahan kualitatif (*relational* dan *extended abstract*), karena detail tersebut terintegrasi menjadi pola struktural.

Kata Kunci: Berpikir Abstrak, Penalaran Abstrak, Pemecahan Masalah, Taksonomi SOLO.

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INTRODUCTION

Research on the issues of developing abstract thinking has been widely carried out, but they have different ways to stimulate the most appropriate solution for development of this thought (Gilead, Liberman, & Maril, 2014; Lermer, Streicher, Sachs, Raue, & Frey, 2016). Abstract thinking level is closely related to students' academic achievement. How students understand and learn depends on cognitive processing ability and abstract thinking level (Darwish, 2014). There had been many studies confirming that the level of abstract thinking predicted students' academic achievement in mathematics and science as well as other fields of science (Gilead et al., 2014; Lermer et al., 2016). In some cases, students' abstract thinking abilities are confronted with cognitive obstacles, didactic, psychological and epistemological obstacles (Komala, 2018). Therefore, students must be conditioned through practice, giving scaffolding and raising awareness of learning through the investigation process.

Discussing about abstract thinking is certainly related to cognitive development. The experts defined cognitive development as a process of acquiring advanced thinking skill and intellectual thinking skill with an ability to use problem-solving approaches in life situations from early to adult age (Gilead et al., 2014; Susac, Bubic, Vrbanc, & Planinic, 2014). Based on the aforementioned discussion, it is deemed necessary to evaluate the abstract thinking that focuses on abstract reasoning process of students at every level of school education.

Abstract Reasoning

Development of students' reasoning abilities should not be considered normal. However, the school must encourage teachers to organize learning that provides opportunities for students to exercise their reasoning skills (Adegoke, 2013). Reasoning is a thinking process arises from emphirical observation resulting in some concepts and understandings. Based on similar observations, proportions will be formed, according to the determined proportions (regarded as correct), a new proportion could be drawn. This process is called as reasoning (Markovits, Thompson, & Brisson, 2015). Abstraction is a construction process in one's mind, which involves reasoning in determining the relationship between mathematical objects and changing this relationship into a specific expression that is independent of mathematical objects (Yilmaz, Argun, & Role, 2018).

Abstract reasoning refers to the ability of information analysis, detecting pattern and relation, and solving problems on complex level (Datta & Roy, 2015). Abilities included in abstract reasoning are: (1) being able to formulate theories about natures of object and idea, (2) being able to understand meanings underlying a happening, statement or object, (3) being able to identify the correlation of verbal and nonverbal ideas, and (4) being able to detect pattern and relation underlying among happening, ideas and objects (Simanjuntak, Abdullah, & Maulana, 2018). Abstract thinking ability is a result of brain maturation (Piaget, 1964). While abstract reasoning is a part of (individual) abstract thinking ability that showing certain abstract thinking level in a certain domain, that will be similarly potential on reasoning ability in other domain (Datta & Roy, 2015).

Piaget introduced four stages of cognitive development that determined reasoning and mental development skills of a person from his childhood to adulthood (Joubish & Khurram, 2011; Piaget, 1964; Simatwa, 2010). In particular, ages 11-14 years old (8th grade students) as an age in which a transition occurs in their cognitive development from stage of concrete operational to formal operational (Susac et al., 2014). Meanwhile, relating to cognitive development, many junior high school students have not yet acquired the ability to understand abstract concepts without a real basis (Darwish, 2014; Simanjuntak, Abdullah, & Maulana, 2018). According to neo-piagetians, a person in charge of abstraction-based reasoning task could be possibly on concrete operational stage while temporarily becomes an expert of formal operational in solving problems based on other kind of abstraction (Joubish & Khurram, 2011).

Profile of cognitive development reveals level of individual abstract thinking skill. This cognitive development level can be measured using a number of intellectual tasks (Ojose, 2008). In this study, the quality of student responses toward a number of intellectual tasks were evaluated with *Structure of the Observed Learning Outcomes* (SOLO) Taxonomy guidance.

The SOLO Taxonomy

The SOLO Taxonomy is an important tool to assess students' knowledge and skills, by examining their answers in depth (Biggs & Tang, 2011; Chalmers, 2011). The assessment in SOLO taxonomy is based on the quality and structure of the answers given by students toward the questions (Korkmaz & Unsal, 2017). By SOLO taxonomy, teachers can identify their answer responses (learning outcomes) so that the students' understanding level toward a given problem can be determined (Özdemir & Yıldırz, 2015).

The SOLO Taxonomy classifies the ability of students' responses to problems into 5 different hierarchical levels:

Level 0 : *Pre-structural*. At this level, students use knowledge without understanding, just repeat the given questions (Goff, Potter, & Pierre, 2014; Potter & Kustra, 2012), and even wrong answers or those of not answering the question (Korkmaz & Unsal, 2017).

Level 1 : *Uni-structural*. At this level, students have a limited understanding. Students focus only on the use of data related to the question (Biggs & Tang, 2011).

Level 2 : *Multi-structural*. At this level, students can focus on more than one aspect to the question but these are not related to each other (Biggs & Tang, 2011; Chalmers, 2011).

Level 3 : *Relational*. At this level, students understand how to build the whole and the relationships between the structures that make up the whole (Biggs & Tang, 2011).

Level 4 : *Extended Abstract*. Students can reason by considering abstract characteristics and can make generalizations. Students can view topics from many perspectives, hypothesize, and make generalizations (Biggs & Tang, 2011; Brabrand & Dahl, 2009).

In content standard of Indonesia curriculum 2013, learning outcomes are grouped into 5 (five) categories based on SOLO Taxonomy: *Pre-structural* (level 0 is Kindergarten class), *Uni-structural* (level 1 is grade I and II), *Multi-structural* (level 2 is grade III and IV), *Relational* (level 3 is grade V and VI), and *Extended Abstract* (levels 4 and 5 are grade VII, VIII, and IX) (BSNP, 2013). Considering this, it is clearly emphasized that, learning mathematics in grade 8, teachers should encourage students to acquire learning achievement on high level thinking pattern or extended abstract.

The description of the learning cycle in the learning achievement of the SOLO taxonomy begins before the cycle, which is pre-structural, in the cycle of the quantitativephase (uni-structural and multi-structural), the qualitative phase (relational), and out of the cycle of qualitative (abstract expanded). The following is presented in Figure 1 of the learning cycle along with the explanation of each level in the achievement of the SOLO taxonomy learning.

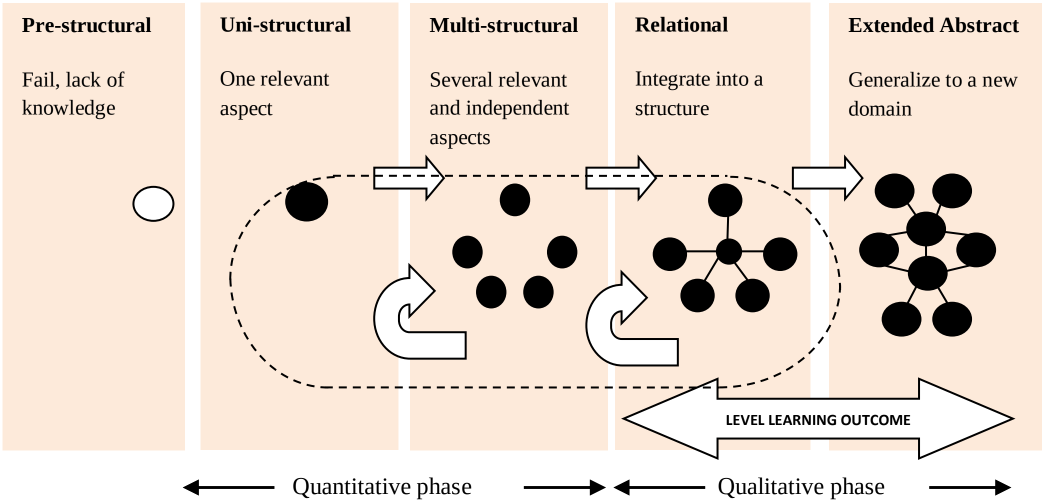


Figure 1. Bigg's SOLO Taxonomy: Learning Outcomes Assessment (Goff, Potter, & Pierre, 2014)

Bases on Figure 1, level of uni-structural and multi-structural response was included to quantitative (concrete) thinking phase. Level of relational and extended abstract response was categorized as qualitative (abstract) thinking phase. (Goff, Potter, & Pierre, 2014). In this study, student answer responses on abstract reasoning ability test were evaluated and analyzed, then the answers would be categorized into one of SOLO Taxonomy levels.

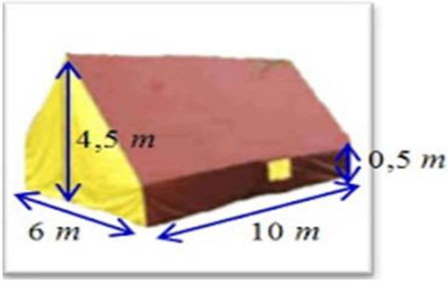
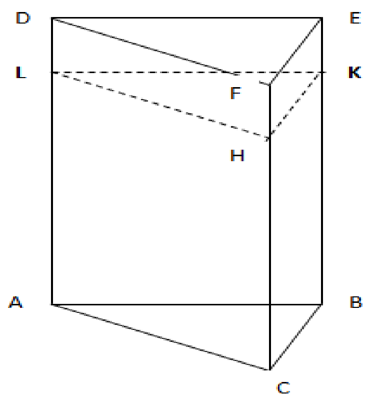
The objective of the study was to analyze and provide an interpretation of students abstract reasoning level in cognitive development based on intended learning outcomes (ILO) guided by Solo taxonomy.The results can be used by teachers as a reference to manage mathematics learning to the level of student cognitive development based on grade level.

METHOD

This research uses quantitative and qualitative descriptive approach (Creswell, 2014). Based on purposive sampling technique, 30 students of 8-E class in SMPN 10 (State Junior High School) Semarang, Indonesia. The subjects were all at the age range of 12 to 14 years old. In this study, mathematics learning was conducted by applying group investigation learning model which was done for five weeks discussing about polyhedra topic. At the end of the lessons, students were assigned to do intellectual tasks in forms of mathematical reasoning tests.

Data were collected by tests, documentation, observation and interview. The test instrument was an intellectual task to assess students' ability in using formal (abstract) reasoning strategy. The test consisted of 2 (two) points of mathematical reasoning problem about polyhedral geometry. The test instrument had been strictly evaluated by a team of experts (instrument validators) in terms of contents, construction, concurrent and predictive validity. Based on test results, the subjects were divided into three groups: the lower group was filled by students with low cognitive ability, middle group was those with moderate cognitive ability, and upper group was those with high cognitive ability.Students' response or answers as the document of intellectual tasks were observed and analyzed by guidance of learning achievements according to SOLO Taxonomy. Data analysis was described qualitatively as a result of learning achievement on students' abstract reasoning. In order to collect deeper information about abstract reasoning, each group selected 2 (two) students to be interviewed.The following is the test instruments used to measure students' abstract reasoning abilities, in Table 1.

Table 1. Test Instruments to Measure Abstract Reasoning

Problem-Solving Task	Intended Learning Outcomes
Problem 1: Abdullah wants to make a tent of fabric with the model and size as shown in the picture below.  How many square of fabric is required by Abdullah to make one tent with its base?	Relational
Problem 2. Take a look at the ABC.DEF prism image below.  The prism contains water as high as CH. Comparison of length CH: HF = 3: 1 The base plane ABC with the elbow at point C. length AC = 8 *dm*, length AB = 10 *dm*, and height AD = 16 *dm*. If the volume of water in triangular upright prisms is transferred into a length beam of 16 *dm* base plane, base width of 6 dm and height of 8 *dm*, then how high is the water in the block?	Extended Abstract

RESULTS AND DISCUSSION

Results

After having a test in form of intellectual tasks, result of student responses or answers were observed and analyzed based on SOLO taxonomy. The observation and analysis on student responses showed a result that is distributed in Table 2.

Table 2. Scores of Student Response on Abstract Reasoning Test

Cognitive Level	Students	Problem Number	Response Rate Answers					Means Score
			PS	US	MS	R	EA	
Lower		1	0	1	5	0	0	63
	6	2	0	2	4	0	0	53
Middle		1	0	0	4	14	0	81
	18	2	0	2	13	2	1	64
Upper		1	0	0	0	4	2	90
	6	2	0	0	1	1	4	92
Total	30	Means	0%	8%	45%	35%	12%	73

Notes: PS : Pre-Structural; US : Uni-Structural; MS : Multi-Structural; R : Relational; EA : Extended Abstract

Based on the result of the test on mathematical reasoning ability (see Table 2), it was obtained an average score of 30 students is equal to 73 with high category. Furthermore, students' abstract reasoning on the lower, middle, and upper group was evaluated more deeply.

Discussion

Lower Group Students

Considering Table 2, students in the lower group with gained average score by 63 on problem solving task number 1 and on problem number 2 by 53. Thus, it could be said that students' mathematical ability in reasoning component was in low category. Based on SOLO taxonomy point of view, students' responses on the problem number 1 and 2 were just at uni-structural (25%) and multi-structural (75%) levels. The following is an example of the results of the response of the students' answers (selected subject) from the lower group, in Figure 2.

Subject (S.12)
Response Answer on Problem 1

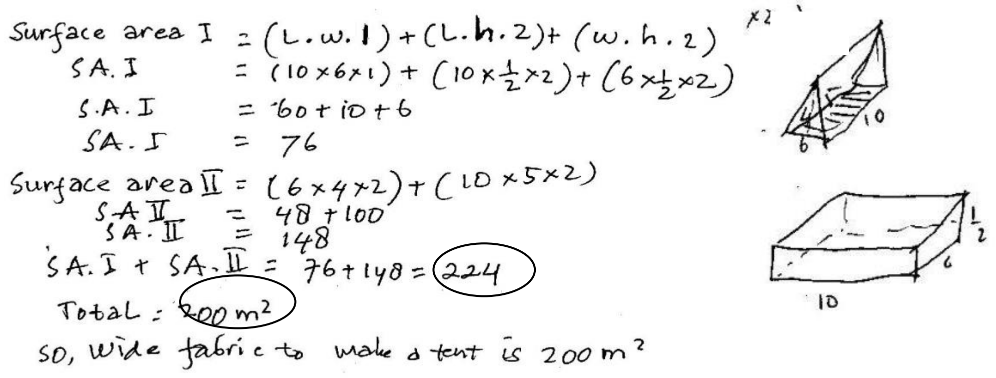


Figure 2. Response of Subject Answers (S.12) in the Lower Group

Referring to Figure 2, it was explained that subject (S.12) could handle various aspects of the topic but could not establish relationships. Subject (S.12) could focus on more than one aspect to the question but could not be related to each other (Biggs & Tang, 2011). Guided by SOLO taxonomy, subject (S.12) answers were at *multi-structural* level. At this level, subject (S.12) were only able to use ideas from concrete instructions to solve problems. Subject (S.12) still thought based on concrete facts and had not been able to establish relationships between aspects of one another. They still operated at the level of quantitative (concrete) thinking and had not yet operated at an advanced stage of cognitive development (qualitative or abstract). Below is an excerpt interview between researcher (R) and subject (S.12).

R	:	Did you use correct way in second step (calculating area II)?
(S.12)	:	I don't know and I don't understand what I wrote.
R	:	You wrote (S.A. I + S.A. II = 224). But your final answer was 200 m^2. Could you please explain your reason?
(S.12)	:	It needed a long time. I knew, and I drew a conclusion that it was right 200 m^2.

Based on analysis on interview result, in this case, subject (S.12) had an obstacle, that they were not able to understand that every concept could have many interpretations. A previous research (Susac et al., 2014) pointed out that many students used a very concrete strategy such as inputing data in form of numbers already in the question. Subject (S.12) could not solve problems in creative ways, and failed in logical conclusion. According to subject (S.12), to apply abstract thinking should take a lot of time. Subject (S.12) answer response in this case is right and correct. However, the subject (S.12) could not provide justification (clarification) or reasons on his answer.

This situation led to a tought that students were difficult to use reasoning based on deduction (Darwish, 2014). Students in the bottom group, have difficulty in abstract reasoning and subject (S.12) are still weak in developing abstraction. In other words, subject (S.12) had not succeeded in developing or improving abstract thinking and logical reasoning.

Based on the result of subject (S.12) assignment in this lower group, teachers need to pay serious attention to the lower group students in mathematics learning. Assistance in learning could be provided by Scaffolding to improve cognitive development to an advanced level (abstract) (Chang, Wang, & Chao, 2009). The hope is not to keep students at a concrete level of thought throughout their study year, which will distract their efforts to solve more complex problem.

Middle Group Students

Regarding to Table 2, students in the middle group obtained an average score on problem solving task number 1 of 81 and problem number 2 of 64. The average score 'moderate' category. Thus, it could be said that the student's mathematical ability in reasoning was in 'fairly good' category. Reviewed by SOLO Taxonomy, students' answers to problems 1 and 2 were at levels of uni-structural (6%), multi-structural (47%), relational (44%), and extended abstract (3%). The following is an example of the results of the response of the students' answers (selected subject) from the middle group, in Figure 3.

Subject (S.08)
Response Answer on Problem 2

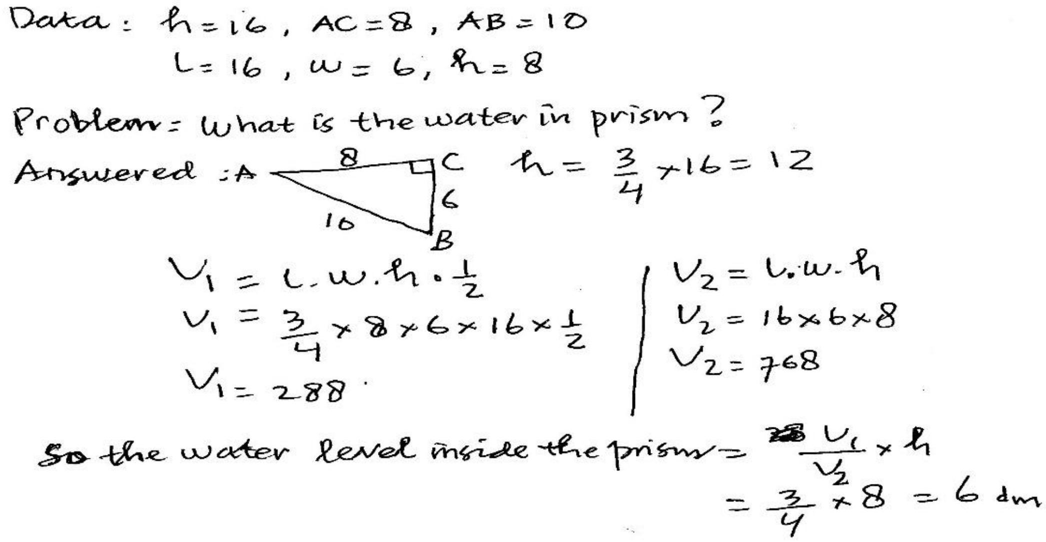


Figure 3. Response of student answers (S.08) in the middle group

Considering Figure 3, subject (S.08) had used steps of problem solving well.However, the subject (S.08) unfortunately failed in decision taking, then the final answer was wrong. Consider the following excerpt of interview between the researcher (R) and the subject (S.08).

R	:	Did you check your answers carefully?
(S.08)	:	I did not do a re-check on my answers.
R	:	What do you know about water volume if it is moved?
(S.08)	:	I did understand. The water volume will be the same although the container is different.
R	:	Your answer is correct, perfect. Please check your answer.
(S.08)	:	It means that the water volume after being moved was 16 x 6 x 6 = 576. Then, the result was not equal to 288 (water volume before movement). So my answer was illogical.

Subject (S.08) could understand the problem in context to triangular prism and moved in context to a cuboid volume. However, subject (S.08) was too soon to draw conclusion without re-checking. So, the answer was wrong and invalid. This shows the subject (S.08) is still weak in identifying the relationship between verbal and nonverbal ideas.

Guided by the SOLO taxonomy, Table 2 shows the responses of students in the middle group being in the quantitative phase of 57% and 43% of them were already operating at the stage of cognitive development qualitative (abstract). But dominantly, students had not been able to see topics from many perspectives, hypothesize, and had not made generalizations. So, cognitive ability had not reached the maximum *extended abstract*. This result might indicate that students were delayed in achieving the expected level of cognitive development of abstract thinking and would develop cognitive abilities as age increased. According to Piaget, at the age of 14, most individuals should be at a formal operational level (abstract level) (Joubish & Khurram, 2011; Mascolo & F., 2015; Piaget, 1964; Simatwa, 2010). Considering this situation, teachers should still believe that students' cognitive development could still be improved through a learning process that focused on improving reasoning ability.

Upper Group Students

Table 2 presented an average score gained by upper group students on problem solving task number 1 was 90 and on problem number 2 was 92. Average score indicated‘very high’ category. Thus, it could be said that students' mathematical ability in the reasoning component was in ‘high’ category. Overviewed by SOLO Taxonomy level, student responses towards problems 1 and 2 were at level of multi-structural (8%), relational (42%), and extended abstract (50%). A total of 92% students in the upper group had operated at the stage of cognitive development qualitative (abstract). The following is an example of the results of the response of the students' answers (selected subject) from the upper group, in Figure 4.

Subject (S.024)
Response Answer on Problem 2

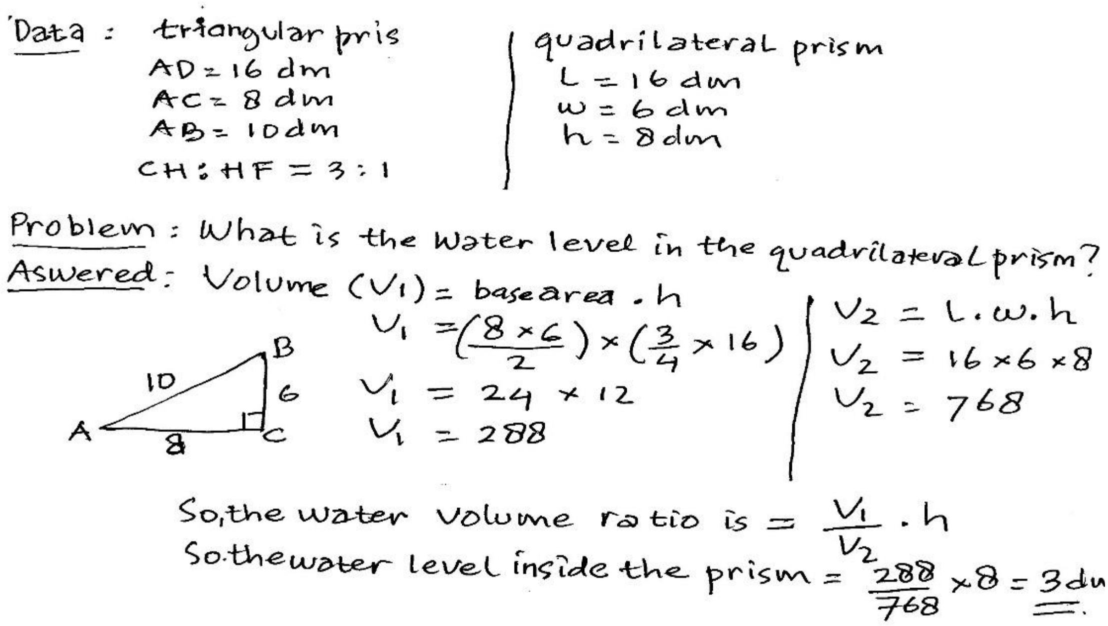


Figure 4. Response of student answers (S.24) in the higher group

On Figure 4, the subject (S.24) had done the steps of problem solving well, systematically, and easily to understand. The subject (S.24) understood how to construct aggregate and correlation among structures that construct the aggregate (Biggs & Tang, 2011). This ability is a high level of abstract reasoning ability. According to Ylvisaker and Hibbard (refered to in Darwish, 2014) abstract reasoning ability relate to the ability of moving what have been learnt from one context to another. The answer by the subject (S.24) question number 2 was said to reach *extended abstract* level. To support this argument, consider the following excerpt of interview between the researcher (R) and the Subject (S.24).

R	:	How did you come up with this answer?
(S.24)	:	I just imagined that the water volume did not change as 288 litres.
R	:	What strategy did you use to solve the problem?
(S.24)	:	I thought that there was a ratio between water volumes (V1) and (V2)
R	:	Is your answer logical?
(S.24)	:	Yes, it is logical. The water volume should not change, it was just the height of water that changed because of different containers.

R : How did you prove that your answer was correct?
(S.24) : I re-checked the steps of problem solving. Then, I formulated volume (V1) and (V2) equation model.

Reviewing Figure 4, it could be described that subject (S.24) could focus on more than one aspect for interrelated questions. Subject (S.24) understood how to build the whole and the relationship between the structures that make up the whole (Biggs & Tang, 2011). Subject (S.24) could reason with considering abstract characteristics and could make generalizations. Subject (S.24) understood that each concept could have many wider conceptual interpretations or conceptual understandings. Subject (S.24) solved problems in a more creative way. Subject (S.24), taking a more complex problem. Subject (S.24) have been able to use abstract things that are not written in direct facts. Subject (S.24) can do the abstraction contained in the problem very well. So it can be said that the subject (S.24) has developed advanced mathematical thinking (Smith, Wigboldus, & Dijksterhuis, 2008). Based on the excerpt of the interview, it can be said that the subject (S.24) has good metacognitive abilities including self regulation and controls the thinking process through repeated checking and reflection (Lukum, Laliyo, & Sukamto, 2015; Qohar & Sumarmo, 2013).

By SOLO taxonomy guidance, in the upper group there is an answer response reaching the 50% abstract extended level, but there are still few students in the quantitative phase is the multi-structural level (8%). Similarly, in middle group students, this result might indicate that some students were delayed in achieving the expected level of cognitive development of abstract thinking and would develop optimally in teen age (Joubish & Khurram, 2011; Mascolo & F., 2015; Piaget, 1964; Simatwa, 2010). Meanwhile, according to Darwish, teenagers gradually developed the ability to use hypothetical-deductive reasoning, and extended their logical thinking to abstract concepts. But this did not mean that there would be no further change in their cognitive (Darwish, 2014). They could seek any excuse, real or imaginary, and had the ability to use scientific reasoning to solve relatively complex problems. This finding shows that abstract reasoning ability plays an important role in the achievement of their mathematics learning outcomes (Widodo, 2017; Yumiati & Noviyanti, 2017).

Overviewing on the results of evaluation on students' abstract reasoning thoughts on the bottom, middle and upper cognitive levels, it could be explained that results of their answers were similar with stages of structure complexity improvement. There were two main changes from concrete thinking to abstract thinking: (1) the quantitative (*uni-structural* and *multi-structural*) stage occured first, as amount of detail in student responses increased and then changed (2) qualitatively (*relational* and *extended abstract*) because the detail was integrated into a structural pattern.

CONCLUSION

Regarding on reviews of previous research result by experts and the results and discussion in this study, it could be said that abstract reasoning of 8th graders was not reaching 100% as expected in content standard of Indonesia curriculum 2013 suggesting that, 8th graders should reach extended abstract level. In the lower group, the abstract reasoning of students was still in phase of quantitative thinking (concrete) with the achievement of SOLO taxonomy was at uni-structural level (25%) and multi-structural level (75%) and no students reached qualitative stage of abstract thinking. Thus, it could be said that lower group students had low abstract reasoning levels. In the middle group, the abstract reasoning of students based on the achievement of the SOLO taxonomy was at an abstract level that extended abstract by only 3%, but by 44% (relational level) they were already operating at the stage of qualitative cognitive development, so the level of abstract thought was quite good. In the upper group, 92% of students have reached the qualitative phase (abstract thinking) which includes 42% at the relational level and 50% of the extended abstract level. Students in the upper group always work with high reasoning and rich abstractions. Basically, result of evaluation on students' abstract reasoning in the lower, middle and upper cognitive level could be concluded that, as student answers' result, it was similar to stages of structure complexity improvement. There were two main changes from concrete thinking to abstract thinking: (1) the quantitative (*uni-structural* and *multi-structural*) stage occured first, as amount of detail in student responses increased and then changed (2) qualitatively (*relational* and *extended abstract*) because the detail was integrated into a structural pattern.

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