JIPM (Jurnal Ilmiah Pendidikan Matematika) 8(2), 2020, 98-107
DOI : 10.25273/jipm.v8i2.5876

JIPM (Jurnal Ilmiah Pendidikan Matematika)
Journal homepage: http://e-journal.unipma.ac.id/index.php/jipm

Alternative in Providing Proof of Negative Exponent for Junior High School Students

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 ISSN 2337-9049 (print), ISSN 2502-4671 (online)

Abstract: This study aims to make a contribution related to opinions and perceptions about negative exponents among junior high school students. In this study, negative exponents are considered as difficult things that require various efforts to instill good knowledge concepts in students. One of them is through evidence in learning mathematics. Evidence is considered capable of instilling students' knowledge concepts in negative exponent material. The research method used is descriptive qualitative research. Descriptive literature research studies through literature reviews, journals, and books relating to negative exponents. The results of this study are alternatives to completing formal and informal evidence related to negative exponents in junior high school students. Through some alternative evidence in introducing negative exponents is expected to make students better understand the form of negative exponents well. So that at the level above students can easily solve problems related to negative exponents without getting errors in the operation of advanced exponents.

Keywords: Negative Exponents; Formal Proof; Informal Proof

Introduction

A number of studies on the level of student understanding of exponential material have been carried out. The results showed that students understanding of exponents in Indonesia still low. Pinahayu (2015, p. 183) said that many people considered exponents to be easy material for students, but the fact found that many students had difficulty in solving exponent problems. Other studies reveal that exponents are material that has complex and difficult concepts (Ulusoy, 2019, p. 2). This is in line with Agustin and Linguistika (2012), and Prastiyowati, Gembong, and Darmadi (2015) who classify exponent material as difficult material, especially in the properties of exponent numbers.

Common mistakes found in students occur in the application of exponents in the form of problems (Ulusoy, 2019). One such error occurs in the operation of negative exponents. For example Angraini and Prahmana (2018, p. 8) in their research found three students had different understandings in solving problems related to exponential traits, specifically negative exponents namely 3^{-5}. The first student answers $3^{-5} = \frac{1}{3^5}$, the error experienced by the first student is a mathematical understanding error which should be $3^{-5} = \frac{1}{3^5} = \frac{1}{243}$. In other words students have not been able to explain the negative exponent problem properly. So the first student's understanding of negative numbers is not entirely correct. For the second student, answer 3^{-5} with the result 243. From the answer, it is known that the understanding of the second student about the negative rank transformed into fractions is still not correct. The third student also does not have a mathematical understanding of the exponent, where the third student has an error by answering 3^{-5} with a result of 0.00003. Likewise in the study conducted by Ulusoy (2019, p. 57) on junior high school students, it was found that students experienced errors in understanding the negative exponent problem given ie 2^{-6}. In the study, the first student interpreted that $2^{-6} = -2^6$ and the second student interpreted $2^{-6} = 2^6$. The first and second students think that the two things are the same, so students' understanding in negative exponents is still wrong.

These errors are obtained because of the lack of students in understanding the concept of negative exponents. Whereas Pianda and Suryani (2018, p. 12) categorize negative exponents into exponential form. Forms of exponents include positive integer exponents, negative integer exponents, and zero exponents (Kemendikbud, 2018). The three forms of exponents begin to learn by students at the junior high level, but at the above level there are still many students experiencing errors in operating exponents, especially negative exponents. Then, how are negative exponents studied at junior high school level?

At junior high school level, the exponent is known as a rank. Rank numbers can be found in mathematics in class IX odd semester. As one of the sub-chapters is a negative rank. At the junior high school level, generally negative numbers are still studied using simple numbers. In the textbook provided by the Ministry of Education and Culture namely Electronic School Book (BSE) class IX mathematics curriculum 2013 revised 2018, negative exponents are defined as $a^{-n} = \frac{1}{a^n}$, where a is a real number instead of zero and n is an integer . To increase students' mathematical understanding, in BSE mathematics class IX there is evidence that helps students understand the concepts of negative exponents. The proof is as follows:

Table 1. Informal Evidence Negative exponents in BSE

Number	Use Rank Division Properties	Conclusion
$\frac{1}{2} = \frac{1}{2^1}$	$\frac{1}{2} = 1:2 = 2^0:2^1$ $= 2^{0-1} = 2^{-1}$	$\frac{1}{2^1} = 2^{-1}$
$\frac{1}{4} = \frac{1}{2^2}$	$\frac{1}{2^2} = 1:2^2 = 2^0:2^2$ $= 2^{0-2} = 2^{-2}$	$\frac{1}{2^2} = 2^{-2}$
$\frac{1}{8} = \frac{1}{2^3}$	$\frac{1}{2^3} = 1:2^3 = 2^0:2^3$ $= 2^{0-3} = 2^{-3}$	$\frac{1}{2^3} = 2^{-3}$

Although there have been alternatives to prove negative exponents, but in the textbook there is only one proof that is related to negative exponents.

The BSE book is the only book given free by the government that can be accessed online on the official website bse.kemdikbud.go.id. So it can be ascertained that the book which only amounted to one is the biggest reference used by many schools in Indonesia. But unfortunately as is known above, in BSE there is only one proof of negative exponents used to instill student understanding. It would be better if students have more knowledge about the alternative proof of negative exponents at the initial level. Because the existence of alternative evidence in introducing negative exponents can make students understand more about the form of negative exponents.

The lack of explanation that can explain the evidence related to negative exponents in class IX textbooks, makes researchers want to provide alternatives related to the proof of negative exponents for junior high school students. By knowing the proof of negative exponents at the junior high school level, students are expected to be able to understand the concept of negative exponents from the start. So in the next level, students can easily solve problems related to negative exponents without making errors in the operation of advanced exponents.

Based on the problems outlined above, the formulation of the problem in this study is provided some alternative how to prove negative exponents at the junior secondary level.

Method

The research method used is qualitative research, with descriptive qualitative research. The purpose of this study is to provide an explanation related to the evidence of negative exponents in junior high school students. Descriptive research chosen is literature study, with a review of several literatures, journals, and books. As Nazir (2003) stated that the technique in collecting data in literature study is a technique by conducting a study of books, literature, notes, or reports that have to do with problems solving related to proof (Suandito, 2017, p. 14).

Data collection is done by searching various sources in the form of books and journals. The first stage, we search for literature through sources including: sinta2.ristekdikti.go.id, garuda.ristekdikti.go.id, scholar.google.co.id, and google.co.id. We conducted a search using the term " exponent " and then several groups of search terms including research on negative exponents, negative exponent content, textbook analysis containing negative exponent material, formal evidence of negative exponents, and informal evidence of negative exponents which is summarized as a key theme in reviewing the literature obtained. A search with the same keywords in English was searched using scholar.google.co.id and 10 international journals in mathematics education submitted by Otten and Nivens (2016).

In the second phase, we looked at textbooks and other books from several literatures such as domestic private publishers, BSE k-13 previous editions, foreign books, university books, and related websites. In the third stage, we systematically check whether the journals, books and websites found identify relevant literature. These journals were chosen based on their scope which included various evidences of negative exponents.

All data in the form of documents obtained are further sorted according to relevant and scientific levels. The data processing techniques are carried out as follows: (1) data is organized and grouped according to categories; (2) test the problem of data taken; (3) analyze the contents in the data. Then for the data analysis process, based on what Raco (2010, p. 123) submitted, the qualitative data analysis process can be carried out in five stages. First read over and over the data obtained by reducing the overlap of information. Second see how important the data obtained. The third classifies data that has compatibility and similarity. Fourth, look for patterns or themes of attachment between one data and another. The fifth is constructing the framework of what is to be conveyed from the data.

With the explanation above, we have selected and collected 25 documents (journals, books and website articles) that have strong relevance. Where 5 documents cover the exponent category, 8 documents cover the category of formal and informal proof in mathematics, and 12 documents about proof of negative exponents. The researcher tests and analyzes data problems that support the research, namely formal and informal evidence in negative exponents, and then ensures that the selected information matches each other, so there is no wrong opinion or response to the subject matter under study. Information data obtained must be written correctly and clearly descriptively. After that, the researcher concludes the results of the research conducted so that the research results found can be easily understood.

Result and Discussion

The curriculum used in Indonesia today is the 2013 curriculum. In the 2013 curriculum learning is required to use scientific methods which include observing, asking, trying, associating, and communicating. Therefore, learns using the 2013 curriculum is expected to be able to improve students' understanding and concepts of knowledge. The process of finding a concept is the most important thing in learning mathematics. Mathematical concepts can be found through reasoning with clear evidence. Proof in learning mathematics is a marker of point understanding of the principles in mathematics. Someone who is able to provide valid proof of one principle in mathematics will be indicated to have a much better understanding than just knowing the meaning and application of a principle (Sumardyono, 2018, p. 510).

A cognitive psychologist named David Paul Ausubel, in the theory of cognitive psychology that he developed said that if a student remembers one thing without linking it to other things, both the process and the results obtained from learning can be expressed as memorization and have no meaning at all to him (Gazali, 2016, p. 186). Therefore, proving a definition or formula in mathematics is not something in the form of memorization but a process by linking several possibilities, so that a strong understanding of the concept is formed. Strong understanding and concepts are needed in mathematics learning both for students and teachers, especially on exponent material.

The exponent is interpreted by Alchian (1960) as a number placed to the right and above the symbol. Meanwhile, according to several other opinions, the exponent is a number that indicates the operation of multiplication repeatedly (Clapham & Nicholson, 2009; Downing, 2009; Negoro & Harahap, 2003). Suppose that a real number and n positive numbers. a^n is the product of the number a as many as n factors that can be written with the equation:

$$a^n = \underbrace{a \times a \times a \times \cdots \times a}_{\text{as many as n factors}}$$

with a as the base number and n as the rank.

Then the equation makes more sense for exponents with positive integers. However, it is not possible to use exponents with negative and zero numbers.

A negative expression is defined by:

$$a^{-n} = \frac{1}{a^n}$$

where a is a nonzero real number and n is an integer.

There are several private textbooks that discuss the concept of negative exponents for junior high school students. In some of these books, some discuss negative exponents very simply and others discuss more abstractly. Understanding the concept of negative exponents in a textbook is discussed in a simple manner, starting with the reasoning of exponential examples from general to specific so that a negative exponent is obtained (Aksin, Ngapiningsih, & Suparno, 2019; Raharjo & Setiawan, 2018; Sembiring, Akhmad, & Nurdiansyah, 2017). The explanation then refers to the formal and informal proof of exponents. Furthermore text books that explain the concept of exponents in the abstract then used as a reference for formal evidence in explaining negative exponents. Meanwhile textbooks that explain negative exponents in simply way used as a reference for informal evidence.

A proof can be in the form of formal and informal evidence (Suandito, 2017). Based on its development, formal evidence is more often used at high school and college levels. Kassios (2009) says that formal proof is not a natural language argument but rather a calculation by following proper rules. Formal evidence is then used as a substitute for using natural (informal) language for several reasons regarding the truth of the statement, using formal notations rigorously, clearly, and can be explained mechanically. This can mean if all the evidence in learning mathematics can be formal unless determined by several other things.

According to what was conveyed by Suandito (Suandito, 2017) The simpler formal evidence is proof using the logic of the premise and axiom set by following the rules of inference while informal proof is proof by everyday words.

Need a process for students to understand the evidence in learning mathematics. The form of representation of evidence adjusts to the development of abilities that develop from enactive evidence, visual evidence, symbolic evidence and formal evidence (Tall, 1992). So the evidence in learning mathematics does not always use formal evidence, sometimes informal evidence is needed to instill strong concepts of knowledge for some students.

The following are formal and informal alternative forms obtained from various available sources:

a. Alternative Formal Forms

Formal thinking ability is generally known as logical thinking ability in the formal-aksimoatik world. Problems will be more acceptable logicly if the solution is accompanied by knowledge of axioms, definitions, lemmas, and theorems (Syawahid, 2015). From this understanding, it is known that formal proof is based on an understanding of axioms, definitions or theorems that already exist.

Based on the definition of numbers of ranks (exponents), it can be seen the properties of the numbers given by Anggraena et al.(2019) including:

a. $a^m \times a^n = a^{m+n}$

b. $a^m : a^n = a^{m-n}$

c. $(a^m)^n = a^{m \times n}$

d. $(a \times b)^n = a^n \times b^n$

e. $(a : b)^n = a^n : b^n$

f. $a^1 = a$

With a rational number not zero and m, n integers.

Therefore, from the characteristics of the exponent, then we can get negative proof of formal form. The formally proof of negative exponents is as follows:

In Dudeja and Madhavi (2018) and Sultan and Artzt (2011) there is a formal proof of negative exponents $a^{-n} = \frac{1}{a^n}$ for $a \neq 0$.

Proof:

Based on the nature of the exponent,

$$\frac{a^m}{a^n} = a^{m-n}$$

Because $-n = 0 - n$, then:

$$a^{-n} = a^{0-n} = \frac{a^0}{a^n}.$$

From the nature of $a^0 = 1$, then: $\frac{a^0}{a^n} = \frac{1}{a^n}$

So $a^{-n} = \frac{1}{a^n}$

In Anggraena et al. (2019) explain the proof of the negative exponent $a^{-n} = \frac{1}{a^n}$ with the properties known above.

Proof:

Based on the nature of b, if $m = 0$, then obtained:

$$a^0 : a^n = a^{0-n}$$

$a^0 : a^n$ can be written as $\frac{a^0}{a^n}$

Because $a^0 = a^n \times a^{-n}$,

Then $a^{-n} = \frac{1}{a^n}$ with $a \neq 0$.

b. Alternative Informal Forms

Ndraha (2015, p. 92) explains informal evidence in mathematics is evidence that not every basic understanding, axioms, definitions, and theorems used are explained explicitly and not every proof step is accompanied by deductive reasons. In learning, both constructive, RME, and curriculum 2013, teachers are required to be able to make students construct their own learning outcomes, so informal media is needed before it becomes formal. The informal media can be in the form of a model or proof without words (Suandito, 2017). So it can be concluded that informal evidence is proof without words or evidence can be said without using theorems or axioms.

In the informal proof of negative exponents namely $a^{-n} = \frac{1}{a^n}$, obtained as follows, among others:

If the positive exponent is multiplying it can be concluded that the negative exponent is allot. Distributing is the inverse or inverse of multiplying. Negative exponents mean how many times to divide that number (Aksin et al., 2019; Dudeja & Madhavi, 2018; Negative Exponents, 2017). Like the following example:

$5^{-3} = 1 \div 5 \div 5 \div 5 = 0.008.$

Or it can be calculated as follows:

$$5^{-3} = 1 \div 5 \div 5 \div 5 = \frac{1}{5^3} = \frac{1}{125} = 0.008$$

Based on the information in Negative Exponents (Negative Exponents, 2017), both positive and negative integer exponents can be explained by using tables to make it easier to understand. A table with examples of the results of exponents of 5 can be presented as follows:

Table 2. Examples of Proof of Negative Exponents

Powers of 5			
	...dst...		
5^2	$1 \times 5 \times 5$	25	↑ 5x Larger
5^1	1×5	5	
5^0	1	1	↓ 5x Smaller
5^{-1}	$1 \div 5$	0,2	
5^{-2}	$1 \div 5 \div 5$	0,04	
	...dst...		

From the table above it can be explained that with the definitions of positive and negative exponents in number 1 the alternative form is informal, if the result of a positive exponent of 5 then 1 is multiplied by 5 over and over according to the number of ranks. But if the exponent is negative than 5 then 1 divided by 5 repeats according to the number of ranks.

The greater the number of positive ranks of 5, the result is 5 times greater. Conversely, the greater the negative rank of 5, the result is 5 times smaller.

In the mathematics book class IX Raharjo and Setiawan (Raharjo & Setiawan, 2018) there is evidence of negative exponents in a unique way, namely by means of mounting between the lines as follows:

Consider the following number correspondence

$$\begin{array}{cccccccccc} 3^3, & 3^2, & 3^1, & 3^0, & 3^{-1}, & 3^{-2}, & 3^{-3}, & \ldots, & & 3^{-n} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & & \downarrow \\ 27 & 9 & 3 & 1 & \ldots & \ldots & \ldots & \ldots & \ldots & \end{array}$$

After paying attention to the pattern, it turns out that in the sequence of numbers the next number is obtained from the previous number divided by 3. Thus, the complete contents of the sequence of numbers are

$$\begin{array}{ccccccccc} 3^3, & 3^2, & 3^1, & 3^0, & 3^{-1}, & 3^{-2}, & 3^{-3}, & \ldots, & 3^{-n} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 27 & 9 & 3 & 1 & \frac{1}{3} & \frac{1}{9} & \frac{1}{27} & \ldots & \\ & & & & \downarrow & \downarrow & \downarrow & & \end{array}$$

Based on these results, in general for integers n and with principal numbers a with a> 0, as well as a ≠ 1 a pattern will be obtained:

$$\begin{array}{ccccccccc} a^3, & a^2, & a^1, & a^0, & a^{-1}, & a^{-2}, & a^{-3}, & \ldots, & a^{-n} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \downarrow \\ a^3 & a^2 & a^1 & 1 & \frac{1}{a^1} & \frac{1}{a^2} & \frac{1}{a^3} & \ldots & \frac{1}{a^n} \end{array}$$

Thus, obtained:

$\mathrm{a}^0 = 1$ and $\mathrm{a}^{-\mathrm{n}} = \frac{1}{\mathrm{a}^\mathrm{n}}$, for n $\in$ primary integers and a > 0 .

Evidence for negative exponents can be explained by means of quotient of numbers (Anggraena et al., 2019; Hall, 2011) as follows:

Find the quotient for $\frac{6^2}{6^5}$.

If you reduce rank,

$$\frac{6^2}{6^5} = 6^{(2-5)} = 6^{-3}.$$

If you write down factors,

$$\frac{6^2}{6^5} = \frac{6\,.\,6}{6\,.6\,.6\,.6\,.6} = \frac{1}{6\,.6\,.6} = \frac{1}{6^3}$$

So $\frac{6^2}{6^5} = 6^{-3}$ and $\frac{6^2}{6^5} = \frac{1}{6^3}$.

Conclusion

The government through bse.kemdikbud.go.id has provided free textbooks from elementary to high school level. The book is called the Electronic School Book (BSE). However, only one kind of odd semester junior high school class book was found. The book contains exponential material including how to prove negative exponents. Unfortunately, there is only one proof of negative exponents. This research provides an alternative proof of negative exponents in the form of formal and informal. By having this alternative proof, teachers and students can use it to make an alternative proof of negative exponents, given that negative exponents are difficult material for students

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