The Mathematics Enthusiast

Volume 15
Number 3 *Number 3*

Article 10

7-1-2018

Two notions of 'linear function' in lower secondary school and missed opportunities for students' first meeting with functions

Dyana Wijayanti

Two notions of 'linear function' in lower secondary school and missed opportunities for students' first meeting with functions

Dyana Wijayanti[1]
Sultan Agung Islamic University, Indonesia

Abstract : *The notion of function is considered one of the most difficult parts of the common lower secondary curriculum. In this paper we discuss the potential role played by linear functions, invariably used as first examples of this new notion. As empirical basis we use a praxeological analysis of the function chapter in four Indonesian lower secondary textbooks. The main point of our analysis is that the class of functions of type* $f(x) = ax$ *(where a is some given number) does not appear explicitly at the level of theory, neither in the sense of being given a name, or in the sense that properties of the class is studied. We discuss the implications of this for students' learning of the more general (theoretical) notion of function.*

Keywords: functions, linear function, praxeology

1. Introduction

It is well known that Klein (1908, 2016) successfully proposed that functions should occupy a fundamental place in school mathematics, and more precisely that

> *We begin with the graphical representation of the simplest functions, of polynomials, and rational functions of one variable* (2016, p. 82).

Sriraman & Törner (2008) claimed that present day emphasis of using functions

[1] dyana.wijayanti@ind.ku.dk

The Mathematics Enthusiast, **ISSN 1551-3440, vol. 15, no.3**, pp. 467-482

(or functional thinking) as a conceptual building block is reminiscent of a pre-existing Meraner Program from 1905 which emphasized functional thinking as a building block for algebra and geometry dating back to Klein's era.

This paper concerns the very first steps of this plan, in its current and potential form as it can be observed in textbooks - and, in particular, the "simplest functions" considered there.
As we shall see in more detail later, in lower secondary school, the notion of function is indeed introduced through the elementary example of first degree polynomials, that is functions of type $f(x) = ax + b$, where a and b are fixed numbers. And the graphical representation is immediately and centrally discussed; it also motivates that such functions are called *linear* in secondary level textbooks.

However, in more advanced (or, in Klein's terms, "higher") mathematics, "linear function" has a different meaning as well. The entry for "linear function" on Wikipedia (Linear function, n.d) reads:

In mathematics, the term **linear function** refers to two distinct but related notions:

- In calculus and related areas, a linear function is a polynomial function of degree zero or one, or is the zero polynomial.
- In linear algebra and functional analysis, a linear function is a linear map.

Thus, the definition which is relevant for "calculus and related areas" is the one given above. It is distinct, and in fact different, from the definition used in linear algebra:

Definition (linear algebra). Consider a map $f: V \to W$ between two vector spaces V and W over the scalar field K. We say that f is *linear* if the following two properties hold:

$$\text{(L1) } f(x+y) = f(x) + f(y) \text{ for all } x, y \in V$$

$$\text{(L2) } f(cx) = cf(x) \text{ for all } x \in V \text{ and all } c \in K.$$

We will need the following, which is well known and easy to prove (only (D)⇒(E) is slightly non-trivial - notice that in the special case of $= \mathbb{R}$, we have also $K = \mathbb{R}$):

Theorem. For a function $f: \mathbb{R} \to \mathbb{R}$ the following conditions are equivalent:

(A) f is a linear map

(B) We have $f(x) = f(1)x$ for all $x \in \mathbb{R}$

(C) There is $a \in \mathbb{R}$ such that $f(x) = ax$ for all $x \in \mathbb{R}$

(D) f is continuous and satisfies (L1)

(E) f satisfies (L2).

We notice also (L1) is *not* sufficient to ensure linearity of a function $f: \mathbb{R} \to \mathbb{R}$ (See for instance Anderson, 1979); but (L2) suffices.

From the condition (C) we also see that the linear algebra notion gives a more restricted class of functions than the definition used in secondary level textbooks.

In this paper, we shall follow the secondary mathematics terminology, so that a *linear function* $f: \mathbb{R} \to \mathbb{R}$ is any function given by $f(x) = ax + b$ for some numbers *a* and *b*[2]. This also gives the simplest class of polynomials (first degree), as prescribed by Klein, and thus a natural first step towards the classes of functions studied in the Calculus.

In the next two sections, we consider more closely the implicit and explicit roles, in the lower secondary curriculum, of the function class given by the conditions (A)-(E) above. Our main point will be that this function class and its properties are not explicitly treated at this level, and that -

[2] This is different from the choice made in De Bock, Neyens, and Van Dooren (2016), namely to use the word affine for this function

given students' familiarity with phenomena and problems which such functions can be used to model - this implies a loss of significant potentials to support the first introduction to the notion of function.

2. Proportion functions

We shall follow Van Dooren, De Bock, Hessels, Janssens, and Verschaffel (2005) and use the term *proportion function* for a function of the form $f(x) = ax$. Roughly speaking, their study demonstrates how students at in lower secondary school tend to use "proportion models" (based on multiplication) even for word problems which call for different models, including linear but non-proportional ones. The "over-use" of proportional models is ascribed to the place tasks requiring such models occupy in the primary and lower secondary curriculum - not as explicit functions (indeed, functions appear relatively late in the curriculum) but as calculations techniques for exercises in which some numbers are given and others are to be found.

The most important of these techniques concern missing value tasks; an systematic study of how these appear in Indonesian textbooks was presented in Wijayanti and Winsløw (2015). These tasks come in various types, but the most common form gives a context which allows to assume the relationship $\frac{a}{b} = \frac{c}{d}$ for four numbers *a*,*b*,*c*,*d* out of which three are given and the fourth is to be computed. Students can solve such tasks by using some "cross product technique"; it invariably asks to multiply the "corresponding" known quantity by a ratio of other known quantities, for instance (to find *c*): $c = \frac{a}{b} \cdot d$. We note that this technique can also be expressed based on a proportion function $f(x) = \frac{a}{b}x$ which, given a number *d*, computes the "corresponding" number *c*. Let us consider a typical exercise:

> If 2 bag can load 4 kg rice, and how much can 5 bags load ?

The unknown weight that 5 bags can load is, with the arithmetical technique, computed as

$$\frac{4\ \text{kg}}{2\ \text{bags}} * 5\ \text{bags} = 10\ \text{kg}$$

while the function approach takes the ratio (4/2 = 2) as the constant to multiply the given number of bags (x) with, in order to get the weight which that number of bags can load:

$$f(x) = 2x.$$

In such tasks, the "constant which defines f" (here 2) can be interpreted using condition (B) in the theorem: namely, it is the value of the unit (here, the weight loaded by one bag).

Proportion functions can also be connected to core material from the lower secondary geometry curriculum. For instance, Wijayanti (2016) found that cross product techniques also appear as central techniques in textbook exercises on similar polygons, and these techniques can be expressed (just as above) in terms of proportion functions which, for two similar polygons *A* and *B*, compute the length $f(x)$ of a side in *B* which corresponds to a given side *x* in *A*. Again the constant defining *f* is the value of the unit, namely the length of a segment in *B* corresponding to a unit length in *A*.

In both cases (as well as in other types of task related to proportion and ratio) it is interesting to consider the meaning of the two linearity conditions, (L1) and (L2). Here, (L1) often has a natural and evident meaning. For instance, in the case of weight held by rice bags, it seems clear that the weight held by 2+5 rice bags must be the weight held by 2 bags, plus the weight held by 5 bags; or that if a side in polygon *A* is divided into two parts of length 2 and 5, then the length of the corresponding side in *B* can be computed as the sum of lengths of segments which correspond to the two parts. (L2), by contrast, may not be so evident. But as continuity can usually be assumed in practical contexts, the theorem above means that "additive functions" are in practice the same as proportion functions".

The above remarks should be sufficient to convince the reader that a separate, explicit discussion of proportion functions and their properties (such as those listed in the Theorem) would be able to draw on, and formalize, considerable parts of lower secondary school students' previous knowledge. By contrast the more general case (of linear functions) relates to much less, essentially the equation of straight lines (if these are studied before functions, which is not always the case as we shall see). For the first meeting with functions, and an experience of functions as meaningful and useful generalization of familiar knowledge, this more general class of "examples" thus presents itself as much less potent. We have illustrated these remarks in Figure 1, where each box corresponds to a more or less heavy "theme" in the curriculum. Jumping over the "proportion function theme" (marked grey), or merely visiting it briefly as an example of linear functions, leads to cutting off the part of the curriculum relying on functions from very important previous themes. Knowledge established there may then subsist and continue to appear in unwanted forms, as shown by the work of Van Dooren et al. (2005) especially if, as is often the case, students do not really become familiar with the independent curriculum (marked by dashed box).

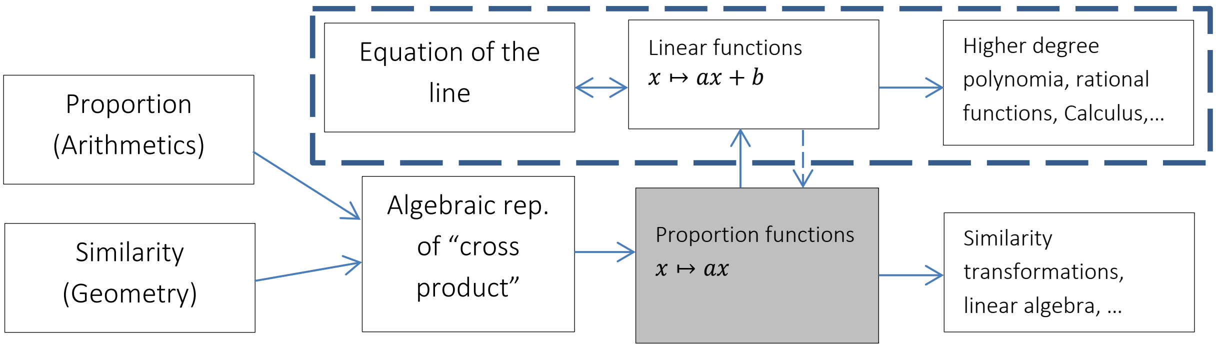

Figure 1. Potential links among themes related to functions

We shall now take a closer look at what is actually the case in a sample of textbooks from Indonesia which are representative of the curriculum in that country (and probably others).

3. Linear and proportion function in lower secondary textbooks

It is required for every public school in Indonesia to provide textbooks for students. To implement this regulation, the government provides a website (http://bse.kemdikbud.go.id/) where students and teachers can download online textbooks for free. Additionally, schools can also use public funding to buy printed copies of those online textbooks. The combination of affordable price, limited educational funding, and government regulation makes most public school use these online textbooks. We focus on the online Indonesian lower secondary textbook for grade 8 (13-15 years old), which is when functions are introduced in the centrally mandated course of study; there is always a chapter entitled "Functions" or similar, and this is what we analyse in this section. Four textbooks has been considered (Nuharini and Wahyuni (2008); Marsigit, Erliani, Dhoruri, and Sugiman (2011); Agus (2008); Nugroho and Meisaroh (2009).

Our analysis is based on praxeologies in the sense of the anthropological theory of the didactic (ATD); cf. (Barbé, Bosch, Espinoza, & Gascón, 2005; Chevallard, 1985, 1988; Chevallard & Sensevy, 2014; Winsløw, 2011). This entails analyzing the textbooks in terms of two interrelated levels : the logos or theory level (theoretical explanations, reasonings, definitions etc. which appear in the text) and the praxis level (the practices which students are induced to and which the theory levels explains and justifies). The praxis level consists of types of tasks (T) and their corresponding techniques (τ), i.e. methods to solve tasks of a certain type; the logos level contains to levels, technology (θ) which is the discourse directly pertaining to explain and justify particular techniques, such as the cross product technique considered in the previous sections; and theory (Θ) which frames and justifies such a discourse, e.g. a definition of what it means for a function to be linear. Together, the practice block (T, τ) and a corresponding logos block (θ, Θ) - which may contain several entries for each four variables - constitute a praxeological organisation. Each of the boxes in Figure 1 can be further modelled in terms of praxeological organisations, and a textbook can be analysed in terms of such a model (as explained and exemplified in Wijayanti, 2016; Wijayanti & Winsløw, 2015).

The previous sections indeed contain theoretical elements about proportion functions, namely a definitions and a theorem; also, we have provided some technological discourse in the discussion of common praxis related to proportion and similarity and considered some of its relations with the theoretical elements. Normally, the technology and the theory about linear functions in lower secondary textbooks will be more informal than what we presented, which is more likely to appear at university level. Major potential gap (or missing link) between earlier parts of the primary secondary curriculum and the new theme on Functions, suggested by the global model in Figure 1, will thus be situated more precisely in the extent to which praxis and logos blocks on Functions are explicitly related to formerly established praxis and logos blocks, for instance on similar polygons or proportion problems. We observe what classes of functions are explicitly defined (theory), what praxis blocks are proposed through examples and exercises, and what technology is offered to support the practices. In particular, we are interested in the properties of functions which are explicitly discussed at the level of theory, and how they appear in tasks. We also analyse some typical tasks that involve linear functions. These tasks can be located in the exercise or in the example; we categorize the tasks in as types of task defined by a common technique. Many techniques are just "shown" by examples and reappear implicitly in exercises drawing on those techniques, which is why this work is analytical and not just observational. We will use this analysis of the praxis level as supplementary data to answer questions about the theory level.

3.1. Linear function in textbook (theory level)

We now analyze four Indonesian textbooks from theory level perspective. In three of them we find the explicit use of the term "linear function", and all of them provide multiple examples. Only two textbooks provide a formal definition of linear functions. For example:

A linear function is a function f on the real numbers that is given by $f(x) = ax + b$, where a, b are real numbers and $a \neq 0$. (Marsigit et al., 2011, p. 51)

Two other textbooks give a slightly more informal definition, with an attempt to suggest that there are other kinds of functions which will be studied later:

In this chapter, the functions that you will learn about are just linear functions, that is $f(x) = ax + b$. You will learn about quadratic function and other polynomial functions in the later classes (Nuharini & Wahyuni, 2008, p. 44).

Interestingly, one textbook does not even introduce the 'linear function' term, but as the other books, it explicitly states the relation of these functions to the equation of the straight line:

The equation $f(x) = 2x + 1$ can be changed into the equation $y = 2x + 1$. The equation also can be seen as straight line, why is it? The equation $y = 2x + 1$ is called a straight line equation. (Nugroho & Meisaroh, 2009, p. 50)

Besides the explicit link with the geometric theme of straight line equations, all text books eventually proceed to quadratic functions. None of them give an explicit definition of proportion functions (even with other names) or point them out as a special case of linear functions.

All textbooks include, in the Function chapter, some discussion of the abstract notion of function, based on naïve set theory. This generally includes mentioning and illustration of the notions of domain, codomain, and range. Then, students are also introduced to the notion of relation and how it differs from the more specialized notion of function. Besides examples of functions given by formula (e.g. $f(x) = 2x + 1$), students are presented with four other ways to represent a function: Venn diagrams with arrows between them, Cartesian graph, tables, and sets of ordered pairs. In some of these cases, linear functions are considered on restricted domains, such as all integers or a finite set of them. On the other hand, linear functions are neither connected to examples of earlier praxis with proportion and similarity, nor discussed in terms of functional equations such as (L1) and (L2) which are, of course, also not valid for all linear functions.

3.2 Linear function in textbooks (praxis level)

We now proceed to classify, into types of tasks, the praxis blocks actually proposed to students in relation to linear functions. Before stating the results of our analysis of all exercises in the books, we consider an example to explain how the analysis was done:

Given function $f: x \to 2x - 2$ defined on integer numbers. Determine a. $f(1)$, b. $f(2)$,
(Agus, 2008, p. 30)

Here, students are given the formula defining function on integer numbers, and they are asked to find the image of two integers under *f*. As variant of this kind of task, students are asked to decide their own integer numbers to find the image of function. Sometimes they are also asked to graph the image of the function, or to represent it in terms of ordered pairs of numbers. In any case, the task is classified as being (or at least containing tasks) of the type

T_1: given a function $f\ (x)$ on integers, find the image of function at specific integers.

τ_1: replace variable in the function expression and calculate the result.

Variants of T_1 include students being given two integers *m* and *n*, and are asked to compute $f(m) + f(n)$.In the next type of tasks, students have to do "the inverse" of T_1, namely solve the equation $f(x) = y$ with respect to *y*:

T_2: Given the closed form expression $f(x) = ax + b$ (where *a* and *b* are given), as well as the image of f. Determine the domain of f.

τ_2 : For each value d in the image, solve $ax + b = d$ with respect to x. The domain is then the set of all solutions obtained.

In contrast from T_1, T_2 are asked student to determine domain of function f(x) and image and expression of f(x) given. A simple algebra is needed to manipulate the equation that is resulted from function and the image of f(x). An example can be seen as follows

Function $\mathrm{h(x) = x \rightarrow 7x + 6. Jika\ h(c) = 27}$. Then determine the value of c …

a. 5 c. 3

b. 4 d. 2 (Marsigit et al., 2011, p. 63)

In T_3, some values of a linear function are given at certain points, and students are asked to determine the correct function expression from a list; the technique τ_3 is to evaluate the expressions at the given points and identify the one whose values match the given values. For example:

The price of a pencil is Rp. 1.200,00, the price of two pencils is Rp. 2.400,00, and the price of 5 pencils are Rp. 6.000,00. Which of the following functions describe this?

a. $f : x \rightarrow 1200x$
b. $f : x \rightarrow 2400x$
c. $f : x \rightarrow 1000x + 200$
d. $f : x \rightarrow 1300 - 100$

(Marsigit et al., 2011, p. 63)

Of course, the above exercise is both overdetermined (three values are given, while two would suffice) and underdetermined (it is not stated that the function must be linear, however students only know that type of function). A more classical (and difficult) variant, then, is:

T_4: For a linear function $x \mapsto ax + b$, where a *and* x are given, along with one value of *f* at a point. Determine (the expression defining) *f*.

τ_4: If we are given that $f(s) = t$, solve $as + b = t$ with the respect to the parameter (a or b) which was not given. Insert it into $f(x) = ax + b$.

This types of task can also be varied a bit, as in the following exercise (which also contains two tasks of type T_1):

Given $f\ (x)\ =\ (x + a) + 3$ and $f\ (2) = 7$. Determine a. The function $f(x)$, b. the value of $f(-1)$, c. the value of $f(-2) + f(-1)$. (Nuharini & Wahyuni, 2008, p. 45)

In T_5, students are asked to substitute an algebraic expression into a function f(x):

T_5: given the algebraic expression of a linear function f (x) and another algebraic expression E (depending on x, so $E = E(x)$). Compute $f(E(x))$.

τ_5 : substitute the algebraic expression in to a function f (x), and simplify the result, for instance collecting alike terms if needed.

Here is a case of a worked example of this type of task:

Given the function $f\ (x)\ =\ 2x - 1$, determine: a. $f(x + 1)$; b. $f(x^2)$ (Nugroho & Meisaroh, 2009, p. 41).

The above types of task are all fairly standard and most appear with several exercises to train the technique. There are very few examples of more theoretical exercises, concerning properties (e.g. functional) of functions, rather than just calculations. Here is one of those rare examples which at least hold some potential in that direction:

Given the function $f(x)\ =\ 2x$ defined on real numbers. Determine if $f(-x) = -f(x\,)$. (Nuharini & Wahyuni, 2008, p. 47)

We notice, though, that nothing is said about the meaning or importance of the “property” (in this case, f being an odd function, the generalization to (L2), or the like). So even this example does not really go much beyond unmotivated algebraic manipulation, let alone add to students’ specific knowledge about functions or some class of functions.

4. Discussion

The preceding analysis indicates some striking characteristics of how lower secondary students first meet the notion of function in Indonesian schools, and probably other countries as well: as a relatively new set of tasks which are mostly of algebraic nature, besides the multiple representations which are involved in some tasks (with the crucial graphical one being, at least in Indonesia, mainly worked with in a separate chapter on line equations). Invariably, functions are, at the first meeting, almost exclusively *linear* functions, and although proportion functions appear as examples, they are never named, and there is no explicit mention on the relations to previous themes in the curriculum - especially similar figures in Geometry, and proportion and ratio in Arithmetic, which however concern essentially proportion (rather than general linear) functions. The few apparent exceptions, like the task of type T_3 cited above (on prices of pencils), are not related to the past theme of proportion at the level of theory, which would require both an explicit treatment of (and probably name for) the class of proportion functions, and a set of convincing examples of how functions cannot only formalize but also unify and facilitate the mathematical modelling of phenomena which exhibit constant relative growth (e.g. distance under constant speed, cost under fixed unit price, etc.).

The overall tendency is that the theme mostly has links to future curriculum, namely, towards important topics such as polynomial equations, the algebraization of Geometry via Cartesian diagrams, and further function classes which will become important in upper secondary school.

These are of course themes which the students do not yet know and so the relations which textbooks can make between them and the new theme of (linear) functions are inevitably implicit, or of little use if explicit.

In most books, linear functions are discussed to some extent and related to first order equations and straight lines in coordinate systems, but unlike the modelling of constant growth and similarity, the notion of function does not add much to these themes (while the diagrammatic representation is of course crucial to the students' later praxis with functions). Indeed, at the praxis level, linear equations is a stepping stone towards quadratic equations which is a main topic toward the end of secondary school, and linear functions form the first step in a series of new functions, with linear functions playing a key role in Calculus. We also note that the seven types of tasks, studied in Function chapter, mainly focus on algebraization.

5. Conclusion

This paper has pointed out one problem (the lack of solid relations between the praxeologies which constitute students' first meetings with functions, and praxeologies which are already familiar to students) and a possible solution: developing an intermediate theme on proportion functions, preceding and preparing the study of linear functions. This proposal can be further motivated by recent empirical research (De Bock et al., 2016) showing that most of students are able to recognize characteristic of representation of proportional, inverse proportional, affine function, but this study still does not answer question on why students have difficulty to distinguish these functions. Here, not only a "name" is needed, but also a thorough study of important properties of proportion functions (like additivity, relation between domain and range, multiple representations etc.) and how these properties can be interpreted in modelling contexts which the students have worked with for years. Teachers and curriculum authors should, themselves, become more familiar with functional properties of proportion functions, as stated in the introduction (Theorem), as a rich source of problems and points to study within this theme.

It is well known that functions constitute a delicate abstraction which many students do never really come to grips with, while they may still succeed relatively well with routinized algebraic tasks as the types presented above (and, of course, more advanced ones in upper secondary Calculus). The reason why the "stepping stone" of proportional functions is underdeveloped in most curricula could well be due, in part, to a lack of terminology, given than "linear" is associated with "lines" and thus reserved to the more general case of functions $x \mapsto ax + b$ (with two parameters).

Acknowledgements

This research was supported by the Ministry of Research, Technology and Higher Education of the Republic of Indonesia under Overseas Postgraduate Scholarships (BPP-LN).

References

Agus, N. A. (2008). Mudah belajar matematika 2: untuk kelas VIII SMP/MTs [*Easy way to learn mathematics 2: for grade IX SMP/MTs*]. Retrieved April 17, 2016 from http://bse.kemdikbud.go.id/

Anderson, J. D. (1979). Non-Linear Additive Functions. *Pi Mu Epsilon Journal*, *7*(1), 7-12.

Barbé, J., Bosch, M., Espinoza, L., & Gascón, J. (2005). Didactic restrictions on the teacher's practice: The case of limits of functions in Spanish high schools *Beyond the apparent banality of the mathematics classroom* (pp. 235-268): Springer.

Chevallard, Y. (1985). *La transposition didactique* (Vol. 95): Grenoble: La pensée sauvage.

Chevallard, Y. (1988). *On didactic transposition theory: Some introductory notes.* Paper presented at the international symposium on selected domains of research and development in mathematics education, Bratislava.

Chevallard, Y., & Sensevy, G. (2014). Anthropological Approaches in Mathematics Education, French Perspectives. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 38-43): Springer Netherlands.

De Bock, D., Neyens, D., & Van Dooren, W. (2016). Students' Ability to Connect Function Properties to Different Types of Elementary Functions: An Empirical Study on the Role of

External Representations. *International Journal of Science and Mathematics Education*, 1-17. doi: 10.1007/s10763-016-9724-z

Klein, F. (1908). *Elementarmathematik vom höheren Standpunkt aus. Teil I: Arithmetik, Algebra, Analysis.* . [Elementary Mathematics from a Higher Standpoint. Vol I: Arithmetic, Algebra, Analysis]. Berlin, Heidelberg: Springer.

Klein, F. (2016). *Elementary Mathematics from a Higher Standpoint. Vol I: Arithmetic, Algebra, Analysis*. (G. Schubring, trans). Berlin, Heidelberg: Springer.

Linear function. (n.d). Encyclopedia of Mathematics. Retrieved January 12, 2017, from https://en.wikipedia.org/wiki/Linear_function

Marsigit, Erliani, E., Dhoruri, A., & Sugiman. (2011). *Matematika 2 untuk SMP/MTs Kelas VIII Mathematics 2 for Junior School Grade VIII* Retrieved from http://bse.kemdikbud.go.id/

Nugroho, H., & Meisaroh, L. (2009). *Matematika 2 : SMP dan MTs Kelas VIII Mathematics 2 for Junior School Grade VIII* Retrieved from http://bse.kemdikbud.go.id/

Nuharini, D., & Wahyuni, T. (2008). Matematika konsep dan aplikasinya: untuk kelas VIII ,SMP/MTs 2 [*Mathematics concept and application for grade VIII, junior high school 2*]. Retrieved April 17, 2016 http://bse.kemdikbud.go.id/

Sriraman,B., & Törner,G. (2008). Poltical Union/ Mathematics Educational Disunion: Building Bridges in European Didactical Traditions. In L.D. English et al., (Eds). *Handbook of International Research in Mathematics Education (*pp. 660-694). Routledge, Taylor and Francis.

Van Dooren, W., De Bock, D., Hessels, A., Janssens, D., & Verschaffel, L. (2005). Not Everything Is Proportional: Effects of Age and Problem Type on Propensities for Overgeneralization. *Cognition and Instruction, 23*(1), 57-86.

Wijayanti, D. (2016). *A reference model for analysing textbook treatment of similarity in plane geometry*. Manuscript submitted for publication.

Wijayanti, D., & Winsløw, C. (2015). *A new approach to textbooks analysis: praxeological reference models, the case of proportion*. Manuscript submitted for publication.

Winsløw, C. (2011). Anthropological theory of didactic phenomena: Some examples and principles of its use in the study of mathematics education. *Un panorama de la TAD. An overview of ATD. CRM Documents, 10*, 533-551.